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PRACTICAL MENSURATION

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Roy Benedict.

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Ontario.

PRACTICAL

MENSURATION

FOR THE USE OF STUDENTS IN
COLLEGES, AND SCHOOLS, AND
FOR PRIVATE LEARNERS.

BY

C. A. FLEMING, C. A.,

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"Commercial Law and Business Papers"; "How to
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PREFACE.

The design of the author of this book is to supply a work on Mensuration suitable for students in Business Colleges, Public Schools and other institutions where Euclid and Algebra are not taught, but where so useful and practical a subject as Mensuration should not be neglected. There is no subject more commonly used by all classes of persons, and there is not one that has received so little attention at the hands of the Educators of our country. There is scarcely a transaction in every-day life to which it does not apply from the purchase and sale of a yard of calico to the great railway contracts that require the time of their execution to be counted in years. In this volume the author has endeavored to combine the theoretical and practical, and to present the work in as reasonable a way as possible. It is not desirable that the student's memory be charged with a great variety of decimals, fractions, formulas, etc. Where such are used in this work one decimal or fraction is made to do service throughout an entire class of operations instead of using many modifications of the fraction or decimal. Chapters on square and cube root are placed at the beginning of the work as these rules are used to a considerable extent in mensuration and are placed near the end of arithmetics and but few students reach them, when in fact they might be readily taught immediately after division. The definitions that follow immediately after the exercises in square and cube root are principally a study of the technical words pretaining to the subject and as such should receive considerable attention from the student so that he may readily understand the examples and exercises throughout the book. The theoretical part is placed in the first part of the work, and common measurements in the second part, these however, may be taught simultaneously or a great many examples in the practical part (part II), may be taught before the theory in the first part with very good results.

Third Edition revised and enlarged.

February 1st, 1902.

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INTRODUCTORY.

A knowledge of Square Root and Cube Root is necessary in Mensuration. These rules, in Arithmetics, are usually placed near the end, and many pupils in our public schools who should study Mensuration, do not get any instruction in them. An introductory chapter on these subjects is indespensable.

Definitions.

1 **The Square Root** of a number is one of the two equal factors of that number. Example :—6 is the square root of 36. $6 \times 6 = 36$.

2 **The Cube Root** of a number is one of the three equal factors of that number. 6 is the cube root of 216. $6 \times 6 \times 6 = 216$.

3 **The Radical** sign $\sqrt{\ }$, when placed before a number, indicates that some root of that number is to be found.

4 **The Index** is a small figure placed above the radical sign to denote what root is to be found. Thus $\sqrt[2]{49}$ means the square root of 49 ; $\sqrt[3]{1728}$ means the cube root of 1728; $\sqrt[4]{81}$ means the fourth root of 81. *When no index is used, it is understood that the square root is to be found. Thus* $\sqrt{64}$ *means the square root of 64.*

5 **The Square** of any number is the result produced by multiplying that number by itself. Thus the square of 5 would be 5×5, or 25.

6 **The Cube** of a number is the result produced by multiplying a number by itself twice. Thus the cube of 3 would be 3 × 3 × 3=27.

7 **Factoring.** To find the square root or cube root of a number by factoring.

EXAMPLE

1 Find the square root of 324.

2)324
2)162
3)81
3)27
3)9
3

Square Root = 2 × 3 × 3 = 18

ANALYSIS.—A square root is the product of two equal factors.

In order to find the square root of a number we must find the value of those factors We find that the prime factors of 324 are 2 × 2 × 3 × 3 × 3 × 3. We can make two groups of them. viz. (2 × 3 × 3) × (2 × 3 × 3), and we find that the factors in each group are alike, and when multiplied together amount in each case to 18. Thus we have two equal factors, which when multiplied together will produce 324, therefore the square root of 324 is 2 × 3 × 3, or 18.

RULE. *Resolve the number into its prime factors. Then take one of every two equal factors and multiply together the numbers taken out. The result will be the square root of the given number.*

EXAMPLE

2 Find the cube root of 3375.

3)3375
3)1125
3)375
5)125
5)25
5

ANALYSIS. — A cube is the product of three equal factors.

We find that the prime factors of 3375 may be arranged in three equal groups, viz., (3 × 5) × (3 × 5) × (3 × 5), each of which, multiplied out, amounts to 15. Thus we have three equal factors, which when multiplied together amount to 3375, therefore the cube root of 3375 is 3 × 5, or 15.

RULE.—*Resolve the number into its prime factors Then take one out of every three equal factors and* *multiply together the numbers taken out. The result will be the cube root of the given number.*

EXERCISES

8 Solve the following problems :—

1. $\sqrt{256}$; $\sqrt{400}$
2. $\sqrt{729}$; $\sqrt{625}$
3. $\sqrt[3]{27}$; $\sqrt[3]{343}$
4. $\sqrt[3]{125}$; $\sqrt[3]{216}$
5. $\sqrt{576}$; $\sqrt[3]{729}$
6. $\sqrt{6561}$; $\sqrt[3]{3375}$
7. $\sqrt[3]{9261}$; $\sqrt{1764}$
8. $\sqrt{2916}$; $\sqrt[3]{35937}$

9 The foregoing method only applies to a limited number of cases. It may be advantageously used in a number that is easily factored. We now proceed to give rules for extracting the square and cube roots of numbers, with factoring, that apply to all cases.

Square Root.

10 General Rule to find the square root of any number.

EXAMPLE

Find the square root of 4225.

```
    | 42,25(65
    | 36
125 |---------
    |  625
    |  625
```

ANALYSIS. Beginning at the right hand side, separate the number 42,25 into periods of two figures each. Next find the greatest square contained in the left hand period of figures. The greatest square contained in 42 is 36. The square root of 36 is 6. This will be the first figure of the root. Place it to the right of a given number, as you do in long division. Draw a perpendicular line to the left of the given number. Square the first figure of the root, 6, and subtract the result, 36, from the first period of figures in the given number. To the remainder, 6, affix the next period, 25, in your given number. Multiply that part of the root already found by 2, and place the result, 12, to the left of the perpendicular line, using it as a trial divisor. 12 is contained five times in 62, therefore 5 will be the next figure in your root. Annex five to the root already found, and also to the trial divisor, thus making it 125. Multiply the complete divisor by the last figure of the root, 5, and place the result, 625, under your dividend. There is no remainder, therefore 65 is, the square root.

RULE.—*Beginning at the right hand side, separate the given number into periods of two figures each.*

Find the greatest number whose square is not greater than the first period of figures in the given number. Place that number to the right of the given number. It being the first figure of the root, square it, and subtract the result from the first period of figures in the given number. To the remainder annex the next period of figures. Double the part of the root already found, and, using the result as a trial divisor, annex your quotient to that part of the root already found, and also to the trial divisor, Multiply the divisor by the last figure of the root already found, and subtract the result from the dividend. To the remainder annex the next period of figures, and proceed as before.

NOTE.—(1). Always annex to the trial divisor the same figure that you annex to the root.

NOTE.—(2). If there is a remainder when the periods of figures in the number are all taken up, annex two ciphers to the remainder and place a decimal point in the root. Then proceed as before.

NOTE.—(3). In extracting the square root of fractions, extract the root of both numerator and denominator.—$\sqrt{\frac{25}{36}}=\frac{5}{6}$

NOTE.—(4). To extract the square root of a decimal. Point off in periods of two figures each, beginning at the decimal point, and going to the right, then proceed as in a whole number. Should there be an odd number of digits, complete the last period by annexing a cipher to it.

EXERCISES.

11 Find the square root of :—

1. 1089.
2. 8796.
3. 625.
4. 2025.
5. 9025.
6. 5625.
7. 7225.
8. 15625.
9. 32041.
10. 14641.
11. 11881.
12. 13225.
13. 94249.
14. 86436.
15. 97344.
16. 46656.
17. 123562.
18. 492804.
19. 994009.
20. 2050624.
21. 29855296.
22. 387420489.
23. 7463824.
24. 3976036.

Find the square root of the following :—

1. $\frac{25}{256}$
2. $\frac{169}{196}$
3. $\frac{625}{2304}$
4. $\frac{576}{18225}$
5. $\frac{1369}{1600}$
6. $30\frac{1}{4}$
7. $17\frac{3}{4}$
8. $69\frac{4}{9}$
9. $\frac{5625}{12321}$
10. $\frac{5184}{8649}$
11. $\frac{14641}{110889}$
12. $9092\frac{17}{49}$

Find the square root of the following numbers that involve decimals :—

1. .0625.
2. .1089.
3. .001225.
4. .9604.
5. .512656,
6. .046656.
7. 62.8.
8. 2.
9. 3.
10. 196.1369.
11. 9.000994009.
12. 431.8084.
13. $125\frac{2}{3}$.
14. 1406.25.
15. $\frac{5}{6}$ to 4 places
16. $\frac{7}{8}$ to 4 "
17. $2\frac{3}{8}$ to 4 "
18. $247\frac{5}{11}$ to 4 "

Cube Root.

12 Find the cube root of 79507

EXAMPLE

$4^2 \times 300 = 4800$	79,507(43
$3 \times 4 \times 30 = 360$	64
$3^2 = 9$	
5169	15507
	15507

ANALYSIS.—Beginning at the units place, separate the number into periods of three figures each. The greatest number, the cube of which is less than the left hand period (79) is 4, the cube of which is 64. Place this 4 to the right of your number as the first figure of the root. Cube 4 and subtract the result (64) from the left hand period of figures (79). To the remainder (15) annex the next period of the number (507), forming a new dividend. Multiply the square of the root already found (4) by 300, and use the result as a trial divisor. The trial divisor ($4^2 \times 300 = 4800$) is contained in 15507 3 times, Put 3 as the next figure of the root. Add together the trial divisor (4800) + 30 times the last figure of the root (3) multiplied by the other figure (4) + the square of the last figure of the root (3) to form your complete divisor, 5169. Multiply this by the last figure of the root and subtract the result from the dividend. There is no remainder, so 43 will be the cube root of 79507.

RULE.—*1. Beginning at the units place, separate the given number into periods of three figures each.*

2. Find the greatest cube contained in the first period to the left, and place its cube root to the right of the given number as the first figure of the root. Subtract the cube of the first figure of the root from the left hand period of figures, and to the remainder annex the next period of figures, to form the dividend.

3. To find the trial divisor, multiply the square of the root already found by 300. Divide the dividend by this trial divisor, placing the result as the second figure of the root.

4. To the complete divisor add to the trial divisor, thirty times the last figure of the root, multiplied by the other figures of the root, and the square of the last figure of the root. Multiply this by the last figure of the root, and subtract the result from the dividend.

5. To the remainder annex the next period, and proceed as before.

NOTE—(1) If a cipher occurs in the root, annex two ciphers to the divisor and another period to the dividend.

NOTE—(2) If there be a remainder, place a decimal point in the root and annex three ciphers; then proceed as before.

EXERCISES.

13 Extract the cube root of :—

1. 64
2. 216
3. 15625
4. 166375
5. 474552
6. 7529536
7. 912673
8. 128024064
9. 3048625
10. 48228544
11. .091125
12. .000097336
13. 9.663597
14. 279.726264
15. 12.812904
16. .015625
17. 435519512
18. $\frac{1000}{1331}$
19. $\frac{216}{42875}$
20. $\frac{13824}{15625}$
21. $39\frac{38}{125}$
22. $2\frac{7}{8}$
23. $11\frac{25}{64}$
24. 18629.
25. 137882920
26. 3336271962
27. 2 to 5 places of decimals
28. 3 to 5 " " "
29. 629.8862972
30. 8818629.823372980

1 cubed = 1
2 = 8
3 = 27
4 = 64
5 " = 125
6 = 216
7 " = 343
8 = 512
9 729
10 " = 1000

⁂

MENSURATION.

Mensuration is the measuring or estimating of the areas of surfaces, the volume of solids, and their linear dimensions.

Definitions.

1 **A Point** is that which has no parts or magnitude, but simply denotes position or place.

NOTE. In working Mensuration, a small dot (.) is generally used to denote a point, but such a dot cannot properly be called a geometrical point, for, no matter how small it is, it must necessarily occupy a certain amount of space, and hence it has magnitude.

2 **A Line** (———) is length without breadth, and is used to denote distance.

NOTE. Here, again, it must be noticed that a line, such as is generally drawn in working Mensuration, is not a geometrical line. Draw a line on paper and you will find that, besides measuring a certain distance in length, it also has breadth—possibly very little, but still it has breadth, whereas, a geometrical line denotes length without breadth,

3 **The Extremities** of a line are points. Lines are of two kinds—straight and curved.

4 **A Straight Line** (———) is one which lies evenly between its extreme points. In other words, it is the shortest distance between two points. It is one that does not change its direction.

5 **A Curved Line** (〰〰) is one that changes its direction at every point.

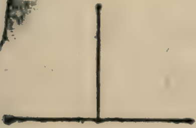
6 A Surface is that which has length and breadth, but no thickness.

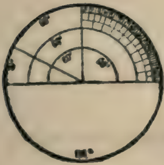
7 A Plane Surface is a flat surface.

8 The Extremities or boundaries of a surface are lines.

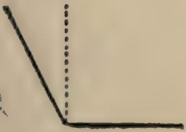
9 An Angle is the inclination of two lines to one another which meet, but are not in the same straight line.

In speaking of an angle it is sometimes designated by one letter, but when two or more angles are together, each of them is designated by three letters, one of which is placed at the vertex of the angle, *i.e.* the place where the lines that form the angle meet, and each of the other two somewhere on one of the two lines containing the angle, the letter at the vertex being placed in the middle in naming the angle.

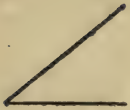




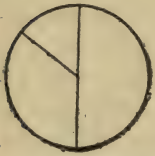
10 When a straight line standing on another straight line makes the adjacent angles equal, each of these angles is said to be a **Right Angle**, and the straight line which stands on the other is said to be perpendicular to it. A right angle contains 90 degrees.



11 An Obtuse Angle is one that is greater than a right angle.



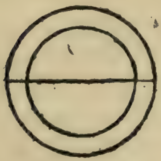
12 An Acute Angle is one that is less than a right angle.



13 A Circle is a figure contained by one line, called the circumference, and is so drawn that all lines drawn from a point within, called the centre, to the circumference, are equal.

14 The Circumference of a circle is the distance around the circle.

15 The Centre of a circle is a certain point within the circle, so situated that all lines drawn from this point to the circumference, are equal.



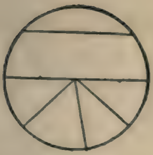
16 The Diameter of a circle is a straight line drawn through the centre of a circle, and terminating at both ends at the circumference.

Note.—It is the longest straight line that can be drawn within a circle.

17 The Radius of a circle is a straight line drawn from the centre to the circumference.

18 A Semicircle is half a circle. It is contained by a diameter, and that part of the circumference cut off by the diameter.

19 A Quadrant (a quarter of a circle,) is that part of a circle contained by two radii at right angles to each other, and that part of the circumference cut off by them.



20 A Chord is a straight line in a circle terminating at both ends at the circumference, but not passing through the centre. (See figure.)

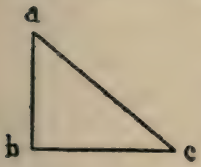
21 An Arc is that part of the circumference cut off by a chord. (See figure.)

22 A Segment is that part of a circle enclosed by a chord and an arc, (See figure)

23 A Sector of a circle is that part contained by two radii and the arc between them. (Any one of the parts on the lower side of the figure.)

24 Rectilineal figures are those which are contained by straight lines.

25 A Triangle is a figure contained or enclosed by three straight sides.

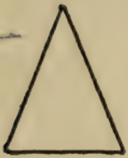


26 A Right Angled Triangle is one which has one of its angles a right angle. The side opposite the right angle is called rhe HYPOTHENUSE, and the sides containing the right angle the BASE and PERPENDICULAR respectively, the base being the line on which it is constructed.

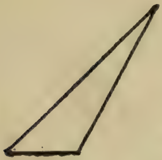
27 An Obtuse Angled Triangle is one which has one obtuse angle.

28 An Acute Angled Triangle is one which has acute angles only.

29 An Equilateral Triangle is one which has all its sides and all its angles equal.



30 An Isosceles Triangle is one which has two of its sides and two of its angles (namely the angles at the base) equal.



31 A Scalene Triangle is one having three unequal sides, and three unequal angles.

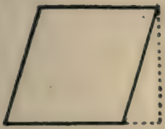
32 A Quadrilateral Figure is one that has four straight sides and four angles.

33 A Parallelogram is a figure having its opposite sides equal and parallel.

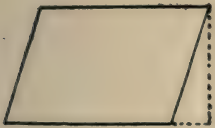
34 A Rectangle is any four-sided figure having its opposite sides equal and all its angles right angles.

35 A Square is a four-sided figure having all its sides equal, and all its angles right angles.

36 An Oblong is a four sided figure having only its opposite sides equal, and all its angles right angles. (The figure given under 34 is an example.)

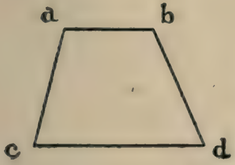


37 **A Rhombus** is a four-sided figure having all its sides equal, but its angles are not right angles.

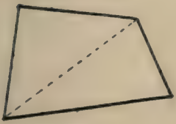


38 **A Rhomboid** is a four-sided figure having its opposite sides equal but its angles are not right angles.

NOTE—In both the foregoing 37 and 38 the opposite angles are equal.

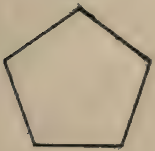


39 **A Trapezoid** is a four-sided figure having only two of its sides parallel.

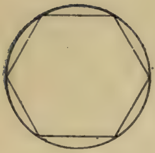


40 All four-sided figures, besides these, are called **Trapeziums.**

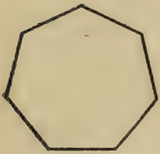
41 Any figure having more than four sides may be call-a **Polygon.**



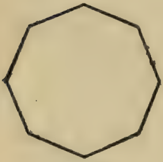
42 **A Pentagon** is a five- sided figure having all its sides equal.



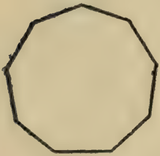
43 A Hexagon is a six-sided figure. (The accompanying illustrates a Hexagon within a circle, showing how it is drawn.)



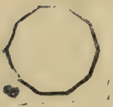
44 A Heptagon is a seven-sided figure.



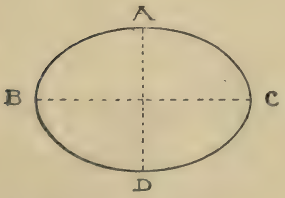
45 An Octagon is an eight-sided figure.



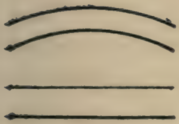
46 A Nonagon is a nine-sided figure.



47 A Decagon is a ten-sided figure.

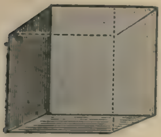


48 An Ellipse is like a circle compressed in one direction and elongated in the other.

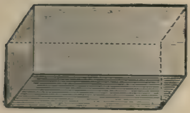


49 **Parallel** lines are lines which have the same direction, and being in the same plane, equally distant from each other, they can never meet.

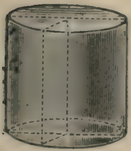
50 **A Solid** is that which has three dimensions, viz., length, breadth and thickness.



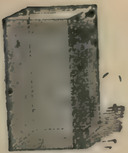
51 **A Cube** is a solid having all its sides equal and all its angles right angles.



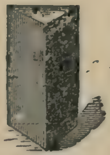
52 **A Parallelopipedon** is a solid bounded by six parallelograms, of which every opposite two are equal and parallel.



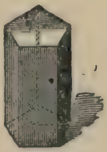
53 **A Cylinder** is a solid having both its ends equal and parallel circles, and bounded by a surface curved similarly to the base.



Quadrangular Prism.



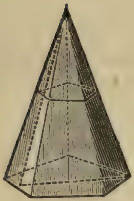
Triangular Prism.



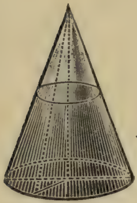
Pentagonal Prism.

54 **A Prism** is a solid, the ends of which are two similar rectangular figures and its sides parallelograms.

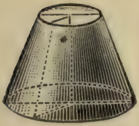
Prisms are called Triangular, Quadrangular, Pentagonal, etc., according to the number of their sides.



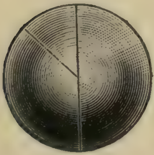
55 A Pyramid is a solid, having for its base a rectilinear figure, and for its sides as many triangles as there are sides to the base; these triangles terminating in one point called the apex or vertex.



56 A Cone is a figure having a circular base and terminating in a point called the apex.



57 A Frustum of a pyramid or cone is that part which remains after the top is cut off parallel to the base.



58 A Sphere or Globe is a figure bounded by a uniformly curved surface, all the points of which are equally distant from a point within called the centre.

TABLES OF MEASURES.

A thorough knowledge of the tables of measures is indispensable to every student of Mensuration. The principal tables are placed here, to be thoroughly learned before continuing the study of this subject.

Table of Linear Measurement

12 Inches (*in*) = 1 Foot (*ft*).
3 Feet = 1 Yard (*yd*).
$5\frac{1}{2}$ Yards = 1 Rod, Pole or Perch (*rd.*, *p.*)
40 Rods = 1 Furlong (*fur*).
8 Furlongs = 1 Mile (*mi*).
3 Miles = 1 League (*lea*).

EQUIVALENTS

1 mi. = 8 fur. = 320 rds. = 1760 yds. = 5280 ft. = 63360 in.
1 " = 40 " = 220 " = 660 " = 7920 "
1 " = $5\frac{1}{2}$ " = $16\frac{1}{2}$ " = 198 "
1 " = 3 " = 36 "
1 " = 12 "

For very small measurements, the line, ($\frac{1}{12}$ inch), is sometimes used. Eighths, sixteenths and thirty-seconds are used more frequently. The decimal parts of an inch are also used.

Surveyor's Linear Measure

100 Links (*l*) = 1 Chain (*ch*).
80 Chains = 1 Mile (*mi*)

EQUIVALENTS

1 mi. = 80 ch. = 320 rds = 1760 yds. = 5280 ft. = 8000 l.
1 ch. = 4 rds = 22 yds. = 66 ft. = 100 l.
1 l. = 7.92 in.

The ordinary roads and streets are 1 Chain wide. This will help to fix on the mind an idea of the length of a chain.

Square Measurement.

144 Square Inches (*sq. in*) = 1 Square Foot (*sq. ft.*)
9 " Feet = 1 " Yard (*sq. yd.*)
30¼ " Yards = 1 " Rod (*sq. rd.*)
160 " Rods = 1 Acre (*a.*)
640 Acres = 1 Square Mile (*sq. mi.*)

EQUIVALENTS

1 sq. mi. = 640 a. = 102400 sq. rds. = 3097600 sq. yds. = 27878400 sq.ft. = 4014489600 sq.in.
1 " = 160 " = 4840 " = 43560 " = 6272640 "
1 " = 30¼ " = 272¼ " = 39204 "
1 " = 9 " = 1296 "
1 " = 144 "

Surveyor's Square Measure

10000 Square Links (sq.l.) = 1 Square Chain (sq.ch,)
10 " Chains = 1 Acre

EQUIVALENTS

1 a. = 10 sq. ch. = 160 sq. rds. = 100000 sq.l.
1 " = 16 " = 10000 "
1 " = 625 "

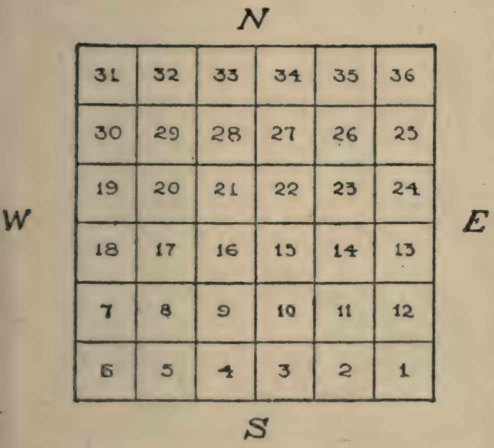
Cubic Measure

1728 Cubic Inches = 1 Cubic Foot
27 " Feet = 1 " Yard
128 " Feet of Wood = 1 Cord

Measurement of Surfaces.

A square surface 1 ft. in width and 1 ft. in length is 1 ft. square, and contains 1 sq. ft.

A square surface 6 ft, long and 6 ft wide would be 6 ft. square, and would contain 6 × 6 = 36 squares, each 1 ft. square, or 36 sq. ft.



31	32	33	34	35	36
30	29	28	27	26	25
19	20	21	22	23	24
18	17	16	15	14	13
7	8	9	10	11	12
6	5	4	3	2	1

You can prove this by drawing a square and dividing each side into six equal parts, each representing one foot. By drawing lines to the opposite dividing points you will find that you will have thirty-six equal squares, each one being a square foot.

A square foot need not necessarily be a foot square. A surface two feet long and six inches wide contains exactly one square foot although not a foot square. Care, therefore, should be taken to distinguish between such expressions as five feet square and five square feet. The former means five feet long by five feet wide, containing twenty-five square feet, while the latter contains only five square feet.

SQUARES AND RECTANGLES.

To Find the Area.

Square

Rectangle

RULE.—*To find area of a rectangle, multiply the length by the breadth.*

NOTE—If length and breadth be given in different terms, (e. g. length in feet and breadth in inches) reduce both to the same terms and multiply them together.

EXAMPLE.—To find the area of a plot of ground 60 ft. × 88 ft. Multiply 60 by 88, and your product, 5280, (88 × 60 = 5280), will be the area of the plot.

EXERCISES.

1. Find the area of a rectangular field 30 × 70 rods.

2. Find the area of a rectangular plot 16 ft. × 10 yards.

3. How many square yards are there in a rectangular lawn 4 rods × 50 feet ?

4. How many acres in a rectangular field 30 rods × 80 rods ?

5. How many acres in a rectangular field 100 yards × 90 rods ?

6. How many acres in a rectangular piece of land 1 mile by 300 rods ?

7. How many yards of carpet will it take to cover a room 13 ft. × 20 ft. ; carpet 1 yard wide ?

8. How many yards of paper will paper a room 10 × 15 ft. ; ceiling 9½ ft. high ? Paper 18 inches wide.

9, Find in acres, etc., the area of a piece of land ⅝ mile long and 800 yards wide.

Rectangle and Square---To Find a Side.

The area and one of the sides of a rectangle being given to find the other.

RULE.—*Divide the area by the given side, and the quotient will be the other side* .

Example.—The area of a rectangle is 156, one of the sides is 12, find the other.

$$\begin{array}{r} 12\overline{)156} \\ 13 \end{array}$$

Since 156 is the product of 12 multiplied by the other side, 156 ÷ 12 will give that side = 13.

EXERCISES.

1. A piece of cloth is 2½ yards wide, and contains 110 square yards. How long is the piece?

2. A piece of road is 22 yards wide, and contains ten acres. How many rods is it in length?

3. A field containing $16\frac{7}{24}$ acres has one of its sides ⅓ mile long. Find the length of the other side in rods.

4. It takes 43½ yards of carpet, one yard wide, to carpet a room, the length of which is 21 ft. Find the width of the room.

5. What is the width of a sidewalk a quarter of a mile long, containing 1173⅓ square yards?

6. One side of a field measures 8 chains. It contains 20 acres. Find the other side.

7. A square field contains 10 acres. Find the length of its sides.

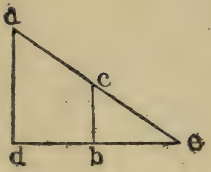
8. The floor of a square hall contains 2671¾ square ft. What is the length of its side.

Note—Extract the square root of the area.

9. A field contains 32 acres and is twice as long as it is wide. Find the length of its side in rods.

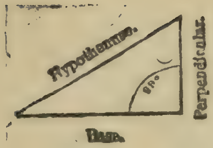
10. A field containing 30 acres is three times as long as it is wide. Find its length and breadth in rods.

TRIANGLES.



Triangles are divided, (see definitions), according to their angles, into three classes, viz., right angled triangles, acute angled triangles, and obtuse angled triangles. Triangles are divided, (see definitions), according to their sides, into equilateral triangles, isosceles triangles, and scalene triangles.

Right Angled Triangles.



To find the Hypothenuse of a right angled triangle, the perpendicular height and the base being given.

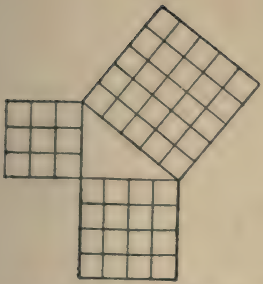
RULE.—*Add together the square of the two sides enclosing the right angle, and extract the square root.*

EXAMPLE

Find the hypothenuse of a triangle having a base 12, and perpendicular height of 16 feet.

12 × 12 = 144
16 × 16 = 256
―――
400 $\sqrt{400} = 20$ = hypothenuse.

20
5



The rule for finding the hypothenuse of a right angled triangle is susceptible of very easy experimental proof. In the annexed figure it will be noticed that we have a right angled triangle having a base 4 ft. long, and a perpendicular of 3 ft. The square constructed on the base contains 4 × 4 = 16 square feet ; on the perpendicular of 3 feet in length is a square containing 3 × 3 = 9 square feet, 9 + 16 = 25 square feet, and this is exactly the area of the square constructed on the hypothenuse, (5 × 5 = 25), hence the sum of the squares of the sides containing the right angle is equal to the square of the side opposite the right angle; and the converse is also true that the square of the side opposite the right angle, minus the square on one of the sides, is equal to the square on the remaining side.

EXERCISES.

Find the hypothenuse of the following triangles :—

1. Base 15 ; perpendicular height 20.
2. " 36 ; " " 40.
3. " 137 ; " " 9.
4. " 6 ft. 3 in. ; " " 3 yds.
5. " 12 ft 5 in. ; " " 6 ft. 3 in.
6. " 16 ft. 8 in. ; " " 103 ft. 6 in.

7. Find the diagonal of a rectangle whose sides are 64 feet and 22½ feet.

8. Find the diagonal of a square whose sides are 15.75 feet long.

9. A smokestack on a sawmill is 60 feet high. What length of wire will I need to fasten to it 8 feet from the top and to reach a stump 70 feet from the base, allowing 9 feet for fastening to stump.

Right Angled Triangles.

To find the Base or Perpendicular in a right-angled triangle, the hypothenuse and one of the sides containing the right angle being given, to find the remaining side.

RULE—*From the square of the hypothenuse subtract the square of the given side, and extract the square root of the remainder.*

According to the foregoing rules, the following propositions are true in a right-angled triangle.

1. $(\text{base})^2 + (\text{perpendicular})^2 = (\text{hypothenuse})^2$.
2. $(\text{hypothenuse})^2 - (\text{perpendicular})^2 = (\text{base})^2$.
3. $(\text{hypothenuse})^2 - (\text{base})^2 = (\text{perpendicular})^2$.

EXAMPLE

The hypothenuse of a right angled triangle is 35, and the base is 28, find the perpendicular height.

$$
\begin{array}{r}
35 \times 35 = 1225 \\
28 \times 28 = 784 \\
\hline
441
\end{array}
\qquad \sqrt{441} = 21 = \text{perpendicular height.}
$$

EXERCISES

Find the bases of the following triangles :—

1. Perpendicular 46 ft. ; hypothenuse 530 ft.
2. " 135 ft. ; " 225 ft.
3. " 2 ft. 8 in. ; " 2 yds.

Find the perpendicular of the following triangles :—

4. Base 9 ft. 9 in. ; hypothenuse 10 ft. 5 in.
5. " 12 yds. ; " 3 rods.
6. " 1 mile ; " 1½ miles.

7. A rectangular field is 60 by 90 rods. Find the distance between its opposite corners.

8. Find the cost of fencing a field in the form of a right angled triangle ; base 25 rods, perpendicular 45 rods ; at 20 cents per yard.

9. Find the diagonal of a plot of ground 1¼ miles square.

10. A boy flying a kite had it caught in a tree. The length of the cord attached to the kite was 250 yards, and he was 240 yards away from the tree. What was the height of the tree ?

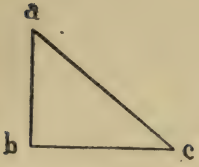
11. What must be the length of a ladder which, when placed 12 feet from a house 50 feet high, will just reach to the top ?

12. A ladder 60 feet long is placed against a building 57 feet high, so as just to reach to the top. How far is the foot of the ladder removed from the building ?

13. The bottom of a ladder is placed in a street 12 feet 10 inches from a building, and the top of the ladder reaches the building 44 feet from the ground, and on turning the ladder over it reaches a building at the other side of the street 36 ft. 8 in. from the ground. Find the width of the street.

14. The span of a roof is 30 feet. The length from the ridge to the eaves is 26 feet. Find the height of the ridge above the eaves.

Area of Triangles.



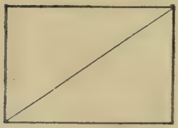
To find the area of any triangle the base and the perpendicular being given.

RULE. *Multiply the base by the perpendicular height and divide by 2.*

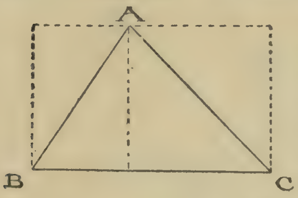
EXAMPLE.

Find the area of a triangle, the base of which is 20 and the perpendicular 25.

$\frac{20\times25}{2} = 250$ sq. ft. area.



NOTE. It is worthy of special notice that a right angled triangle is simply half of a rectangular parallelogram, or half a square. Hence the rule to multiply the length by the breadth (base by the perpendicular,) would give the area of the whole figure; divide by 2 and you have the area of half of it, or the right angled triangle. The same applies to the triangle in the second illustration; and further, every triangle is the half of a parallelogram.



EXERCISES.

Find the areas of triangles having the following dimensions:

1. Base 6 feet; perpendicular height 12 feet.
2. " 3 ft. 7 in. " " 10 ft. 2 in.
3. " 7 yds. 1 ft. " " 1 rod.
4 " 1 mile " " 371 yrds. 2 ft. 8 in.

5. How many acres are there in a triangle, the base of which is 40 rods and the perpendicular 36 rods?

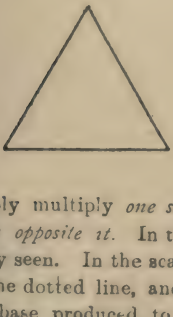
6. Find the area of a right angled triangle having a base of 60 feet, and a hypothenuse of 100 feet.

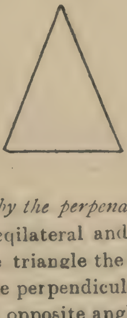
7. The perpendicular of a right angled triangle is 16 feet, and the hypothenuse is 20 feet. Find the area.

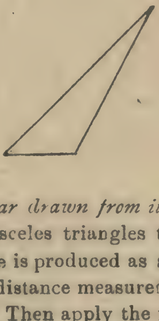
8. The base of a triangle is 32 feet, and the hypothenuse 40 feet. Find the area.

APPLICATION OF RULE TO ANY TRIANGLE.

NOTE. 1. Special attention is necessary to the wide application of this rule. It will be noticed that not only will it apply to right angled triangles, but to all triangles equilateral, scalene, isosceles, etc.







Simply multiply *one side by the perpendicular drawn from it to the angle opposite it.* In the eqilateral and isosceles triangles this is easily seen. In the scalene triangle the base is produced as shown by the dotted line, and the perpendicular distance measured from this base produced, to the opposite angle. Then apply the rule—the base multiplied by the perpendicular height divided by two, equals area.

9. Find the area of an equilateral triangle if the base is 200 feet and the perpendicular height 171 feet.

10. Find the area in acres, &c., of an isosceles triangle, the base of which is 60 rods, and the perpendicular distance to the angle opposite, 40 rods.

11. The base of a scalene triangle is 40 feet, and the perpendicular from the base produced, to the opposite angle is 52 feet. Find the area.

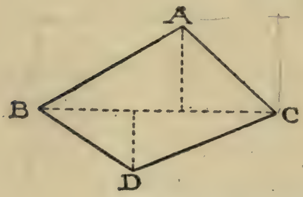
12. The base of a triangle is ¼ of a mile, and the perpendicular distance to the angle opposite is ⅜ mile. Find the area in acres, etc.

13. The area of a triangle is 2480 square feet, the perpendicular distance from the base to the angle opposite is 62 feet. Find the base.

APPLICATION OF RULE TO ANY FIGURE ENCLOSED BY STRAIGHT LINES.

Note. It will be noticed that this rule is also applicable to the measuring of any figure enclosed by straight lines, as any such figure is divisible into triangles.

EXAMPLE.



Find the area of this trapezium. The line joining opposite corners is 42 feet; the perpendicular from this to one angle is 18 feet and to the other angle 16 feet.

The area of upper triangles = $\frac{18\times24}{2}$ = 378

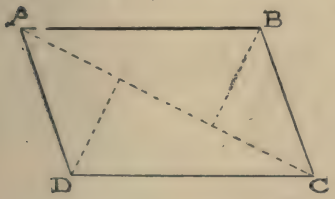
The area of lower triangle = $\frac{16\times24}{2}$ = 336

The sum is the area of trapezium 714

EXERCISES.

14. Find the area in acres of a trapezium, the diagonal of which is 60 rods, and the perpendicular distance to the two opposite angles 30 and 45 rods respectively.

15. Find the area of a trapezium, the distance between two opposite angles being 45 feet, and the perpendicular distance to the remaining angles from this line being 32 and 40 feet respectively.



16. Find the area of this rhomboid in acres, the dimensions given being as follows: diagonal 125 rods, the perpendicular distance to the angle on each side being 40 rods.

To find the Area of Triangles, the Sides being given.

To find the area of any triangle, the three sides being given.

Rule.—*From half the sum of the sides subtract each side separately. Multiply together the three remainders and half the sum of the sides, and extract the square root.*

EXAMPLE.

Find the area of a triangle the sides of which are 30, 40, and 50 feet respectively.

$$30 + 40 + 50 = 120 \div 2 = 60$$

$$\left\{\begin{array}{l}60-30=30\\60-40=20\\60-50=10\end{array}\right.\qquad\begin{array}{l}30\times 20\times 10\times 60=360000.\\ \sqrt{360000}=600\text{ square feet}=\text{area.}\end{array}$$

EXERCISES.

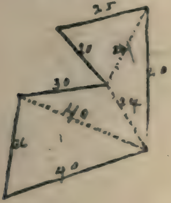
Find to three decimal places, the areas of the triangles having the following sides.

1. 15, 20, 25.
2. 18, 20, 24.
3. 13, 13, 14.
4. 2, 12, 13.
5. 6, 5, 4.
6. 2 rods, 1 rod, 12 yards.
7. 1 mile, 1700 yards, 6 furlongs.
8. 43 yards, 875 yards, 888 yards.
9. 3501 feet, 3604 feet, 3605 feet.

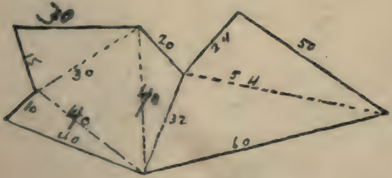
10. The sides of a triangle are 136, 154 and 150. A line is drawn across the triangle and parallel to the longest side, and dividing each of the other sides into two equal parts. What is the area of each of the two parts into which the triangle has been divided?

11. The sides of a triangle are 9 ft. 3 in., 14 ft. 8 in., and 14 ft. 7 in. Two lines are drawn across the triangle parallel to the longest side and dividing the other two sides into three equal parts. Find the area of the parts into which the triangle has been divided.

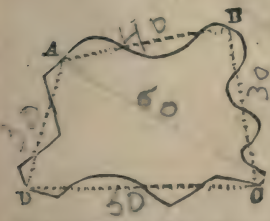
Note 1.—The student should give special attention to this rule for measuring triangles. It is a most useful rule for all classes of measurements of land, etc. The most irregular figures bounded by any number of sides that are straight lines, can be readily measure as they can be divided into triangles by simply joining the angles Find the area of each triangle separately and add them together. In measuring land broken by lakes, rivers, etc., the most difficult piece can be measured without any instrument for striking angle —a simple tape line or measuring pole being sufficient apparatus.



12. Find the area in acres of the annexed irregular field, the dimensions being in rods. 9.308 acres



13. Find the area of the field in the annexed diagram. The dimensions given are rods. 17.174 acres

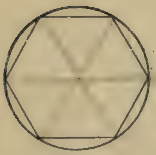


NOTE 2. The use of this rule might be further illustrated by applying it to irregular figures, not having the boundaries straight lines. This is best illustrated by the accompanying drawing. The measurement obtained in this way is simply an approximate, the correctness depending on the judgment of the person making the estimate. Draw the lines in such a way as to leave as much of the surface of the space to be measured outside of the straight lines as there is taken in of the adjoining surface within the lines. The greater number of lines, the greater the accuracy. This is the method employed by surveyors and others in measuring lakes, and land bordering on lakes, &c.

In the above figure measure distance from A to C, then calculate area of triangles, and add together.

14. If the side A B is 40 rods, B C 30 rods, C D 50 rods D A 30 rods, and the diagonal from A to C 60 rods, find, the area of the figure.

THE HEXAGON.



The hexagon is a figure that can easily be drawn and measured.

A hexagon may be constructed by taking a compass and drawing a circle with it; place one point of the compass on the circumference and measure off the length of the radius six times on the circumference. It will be found that the six lengths of the radius can be placed inside the circle. Join these points in the circumference with each other and the hexagon is complete. Join the six angles to the centre from which the circle is drawn and it will be noticed that the hexagon is composed of *six Equilateral Triangles.*

RULE. *Find the area of one of the six triangles into which a hexagon may be divided by the rule on page 37, which reads "from half the sum of the three sides, &c.," and multiply by six.*

EXERCISES.

1. Find the area of a hexagonal lawn having the side ten rods long.

2. Find the area of a hexagon, the side of which is nine feet in length.

3. A garden plot is hexagonal in form, sides 30 feet, except that two of its opposite sides are 120 feet, making it oblong in shape. Find the area, the perpendicular distance between the long sides 51.960.

4. Find the area of a hexagonal oblong having four of its sides 60 rods and the two remaining opposite sides 100 rods, the perpendicular distance between the opposite sides being rods, 103.92.

Rhombus and Rhomboid.

In these figures the student may find difficulty sometimes by proceeding to measure them as rectangular parallelograms. An experimental proof is very easily made, as follows: Take four pieces of wood of equal length. Arrange them in the form of a square and put one nail in each corner, so as to form a flexible frame. It will be noticed that when the angles are right angles that the area enclosed by the frame is *greatest*, and that the area gradually becomes less as the frame is elongated by pushing two opposite angles toward each other. This may be continued until the adjacent sides of the frame will coincide, and therefore not enclose any space.

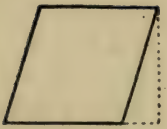
From this experiment it may be gathered:—

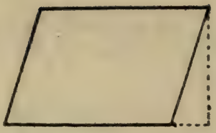
1. That in a four-sided figure when the angles are right angles, it has the greatest area that can be enclosed by those four straight lines.

2. That the area of the four-sided parallelogram becomes less as two of its angles become greater, and the other two less than right angles.

3. That the rule for rectangular figures will not apply.

4. That they must be measured by some such rule as a triangle is measured.





It will be noticed by the accompanying figures that if a piece were cut off one end and placed on the other, that a square would be formed from the rhombus, and a rectangle from the rhomboid, and that the figures thus formed may be measured by the length multiplied by the breadth. The breadth in this case being the *perpendicular distance between the sides*. Hence we have the rule for finding the area of a rhombus and rhomboid.

RULE. *Multiply the base by the perpendicular distance to the opposite side.*

EXAMPLE.

Find the area of a rhombus, the side of which is 24 feet, and the *perpendicular* distance to the opposite side 20 feet.

20 × 24 = 480 square feet.

EXERCISES.

1. The sides of a rhombus are 60 feet, and the perpendicular between the sides 36 feet. Find the area.

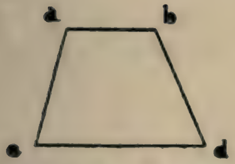
2. The sides of a rhombus are 40 rods, and the perpendicular between them 24 rods. Find the area in acres.

3. The sides of a rhomboid are 3025 feet, and the perpendicular 1080 feet. Find the area in acres.

4. The area of a rhombus is 10 acres, one of the sides is 80 rods. Find the perpendicular. 20 rds.

5. The area of a rhomboid is 6 acres, and the perpendicular distance between the sides 24 rods. Find the side.

THE TRAPEZOID.



The trapezoid is a four-sided figure, having two sides parallel. If a line were drawn through the middle of the sides that are not parallel, at right angles to one of the parallel sides, a piece would be cut off each end, which, if placed on the opposite side, would make the figure into a rectangular parallelogram. The length of the parallel sides would be half the sum of the two parallel sides. Hence the rule :—

RULE. *Add the parallel sides; divide by 2, and multiply by the perpendicular distance between them.*

EXAMPLE.

Find the area in acres of a trapezoid, the parallel sides of which are 20 rods and 30 rods respectively, and the perpendicular distance between them 24 rods.

20 + 30 = 50 ÷ 2 = 25 average length.

24 × 25 = 600 square rods. 600 ÷ 160 = 3¾ acres.

EXERCISES.

1. Find the area of a trapezoid 60 feet and 50 feet long on the parallel sides, and 30 feet perpendicular distance between them.

2. A tapering field is 80 rods long, and the ends which are parallel, are 30 and 50 rods respectively. Find the area in acres.

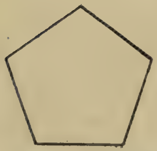
3. A board is 12 feet long, 14 inches wide at one end and 6 inches at the other. Find its area in square feet.

4. Find the area of a board 16 feet long, 18 inches wide at one end, and 12 at the other.

5. Find the area in acres of a trapezoid, the parallel sides of which are 180 and 220 yards, and the perpendicular distance between them 121 yards.

The Trapezium,

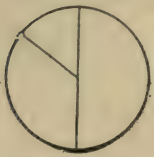
AND OTHER FIGURES.



The trapezium and other irregular figures contained by straight lines are treated under the heading of triangles—page 36 and 38, and as all such figures are easily divisible into triangles they can be measured by either rule. The annexed diagram of a pentagon illustrates the process of dividing regular figures into triangles for convenient measurement.

CIRCLES,

A circle is a plane figure bounded by one line of such curvature that all lines drawn from the centre to it are equal.



A line drawn from the centre to the circumference is called a radius, (plural radii.)

A line drawn from one point in the circumference to another through the centre, is called a diameter.

NOTE. The diameter is equal to two radii.

CIRCULAR TABLE.

60 seconds, marked ″ = 1 minute, marked ′
60 minutes = 1 degree, marked °
360 degrees = 1 circle.

EQUIVALENTS.

1 circle = 360° = 21600′ = 1296000″
1° = 60′ = 3600″
1′ = 60″

Circles—The Circumference.

To find the circumference of a circle, the diameter being given.

RULE. *Multiply the diameter by* $3\frac{1}{7}$, *(or more correctly by* 3.1416.*)*

NOTE 1. There are various modifications of the number $3\frac{1}{7}$ ($\frac{22}{7}$) and of the decimal 3,1416, that are sometimes given in rules for measuring circles. We have, however, been careful to use only the one fraction, or one decimal, to avoid burdening the memory with many decimals and fractions, Some of them would surely be forgotten or misapplied; and further, by the use of one the mind is led to trace the relation between the various operations much better than if separate decimals were given for each operation.

NOTE 2. The fraction $\frac{22}{7}$ ($3\frac{1}{7}$) is not exactly correct, but is near enough for all pratical purposes. By reference to the fifth question following this, it will be seen that it makes a difference of about ten miles in the circumference of the earth, the $\frac{22}{7}$ ($3\frac{1}{7}$) being too great.

NOTE 3. By experiment it has been found that the decimal 3.1416 is also a little too large, and that 3.14159 is a little nearer correct as expressing the relation between the diameter and circumference. This is only necessary in very fine calculations.

EXAMPLE.

Find the circumference of a circle having a diameter of 35 feet.

$$35 \times 3\tfrac{1}{7} = \frac{35 \times 22}{7} = 110 \text{ feet.}$$

EXERCISES.

1. Find the circumference of a circle 42 yards in diameter.

2. A circle is one mile in diameter. Find the distance around it.

3. Find the distance around a circular race course 126 yards in diameter.

4. A lawn in the form of a semicircle is to be fenced, a a cost of 30 cents per yard. Find the total cost, if the diameter of the circle is 168 feet.

5. Find the distance around the earth, if the diameter is 8000 miles, using separately the fraction $\frac{22}{7}$, ($3\frac{1}{7}$) the decimal 3.1416 and the decimal 3.14159. Note the difference in your answers.

Circles—The Diameter.

To find the diameter of a circle from the circumference,

RULE. *Divide the circumference by* $\frac{22}{7}$, ($3\frac{1}{7}$), (*or more correctly,* 3.1416)

EXAMPLE.

The circumference of a circle is 110 miles. Find the diameter.

$$110 \div 3\tfrac{1}{7} = \frac{110 \times 7}{22} = 35 \text{ miles.}$$

EXERCISES.

1. The circumference of a circle is 88 feet Find the diameter.

2. The circumference of a circular lawn is 242 feet. Find its diameter.

3. A circular race course is 891 yds. 1 ft. in circumference. Find its diameter.

4. A circular cistern is 22 feet in circumference. Find its diameter, by both fraction and decimal methods.

5. A circular field is fenced at the rate of 25 cents per rod, at a cost of $66. Find the diameter of the field.

6. A circular lake is 220 miles in circumference. Find its diameter.

Circles---- The Area.

To find the area of a circle, the radius being given,

RULE. *Square the radius and multiply by* $\frac{22}{7}$ ($3\frac{1}{7}$); *or, more accurately, by* 3.1416.

EXAMPLE.

Find the area of a circle having a radius of 35 feet.

$$\frac{35 \times 35 \times 22}{7} = 3850 \text{ sq. ft.} = \text{area.}$$

NOTE. It will be noticed that in finding the area of anything, two dimensions are always multiplied together. In the square or oblong, two sides are multiplied together. In the circle two radii are multiplied together.

EXERCISES.

1. Find the area of a circle having a radius of 42 feet.

2. Find the area of a circular fish pond that is 14 yards in diameter, in square yards.

3. Find the cost of sodding a semi-circular plot of ground the radius of which is 27 feet, at 35 cents per square yard.

4. Find the cost of sodding a semi-circular lawn, the radius of which is 63 feet, at 10 cents per square yard.

5. Find the area of a circle 280 feet radius, by decimal as well as fractional method.

6. Find the area of a circular fish pond 105 feet in diameter.

7. Find the cost of cementing the bottom of a circular cistern 7 feet in diameter, at 4 cents per square foot.

8. Find the cost of the stained glass for a circular window, 10 ft. 6 in. in diameter, at 32 cents per square foot. No allowance for sash.

9. A ventilator in the ceiling of a hall is 21 inches in diameter. One-third of the space is taken up by an ornamental iron screen. Find the actual ventilating surface.

10. Find the area of the end of a barrel 2 feet in diameter, by both fractional and decimal methods.

11. Find the value of a circular island 1 mile 6 furlongs in diameter, at $4 per acre.

12. Find the area of a circular race track 140 rods in diameter.

Circles----Find the Radius.

To find the radius of a circle the area being given:

RULE. *Divide the area by* $\frac{22}{7}$ ($3\frac{1}{7}$), *or* 3.1416, *and extract the square root.*

NOTE. This rule is simply the converse of the preceding rule, hence the operation is just reversed.

EXAMPLE.

Find the radius of a circle having an area of 5544 square feet.

$$5544 \div \tfrac{22}{7} = 1764 \quad \sqrt{1764} = 42 = \text{radius.}$$

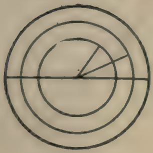
EXERCISES.

1. Find the radius of a circle having an area of 246400 square feet.

2. Find the radius of a circle having an area of 1386 square feet.

3. Find the radius in rods of a circular field containing $31\frac{3}{7}$ acres.

The Area of Rings.



In finding the area of a ring it will be noticed that it is formed by one circle within another drawn from the same centre. These are sometimes called Concentric Circles.

RULE. *Find the area of each circle, and subtract the lesser from the greater, the difference will be the area of the ring.*

NOTE 1. The above rule may be shortened by finding the difference of the squares of the radii of the large and small circles, and multiplying by $\frac{22}{7}$

NOTE 2. The student who has studied algebra will find the above a very nice exercise. Let x = the radius of large, and y of small circle. $x^2 - y^2 = (x - y)\ (x + y)$. Hence the rule may be again modified to the following : ***Multiply the product of the sum and difference of the radii by $\frac{22}{7}$.***

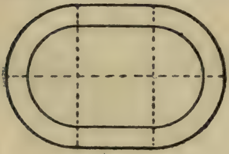
EXERCISES.

1. Find the area of a ring, the radius of the inside being 14 rods, and of the outside circle 21 rods.

2. A circular race track has a radius of 20 rods for the smaller circle and 30 rods for the outer circle. Find the area.

3. Find the area of a ring, the radius of the inside being 40 rods, and the outside being 60 rods.

4. Find the area of a race track as per diagram, greatest length 60 rods, width 40 rods, width of track 8 rods.



5 Find the area of a half mile race track half a mile in length around the middle of the track, width of track 6 rods. The length to be 20 rods greater than the width.

Parts of Circles—The Sector.

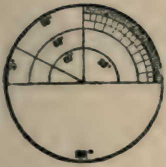
A Sector of a circle is bounded by two radii and the part of the circumference between them. A sector may be greater, equal to, or less than a semicircle.

To find the area of a sector, the radius and the part of the circumference being given:

Rule. *Multiply the radius by the part of the circumference enclosing the sector and divide by two.*

Note 1. It will be noticed that this rule is identical with that for finding the area of a triangle. If a circle were divided into an indefinite number of sectors the curvature would be so imperceptible that infinitely small sectors could not be called any thing but triangles.

This rule is found to be accurate by experimental proof.



Note 2. When the number of degrees are given between the radii, also the length of the radius, the area of the sector may be readily found, there being 360 degrees in a circle. If, for example, the sector contained 30 degrees, then the sector would contain $\frac{30}{360}$ or $\frac{1}{12}$ of the area of the circle.

Rule 2. *Find the area of the circle, multiply by the degrees in the angle of the sector and divide by 360.*

EXAMPLE.

Find the area of a sector of a circle, the radii being 10 feet long and the angle 35°.

$$\frac{10\times10\times22\times35}{7\times360} = 30\tfrac{5}{9} \text{ square feet.}$$

EXERCISES.

1. Find the area of a 90° sector of a circle having a radius of 35 rods.

2. Find the area of a sector of a circle, the part of the circumference being 60 feet, and the radius 45 feet.

3. Find the area, in acres, of a 75° sector of a circle, having a radius of 40 rods.

4. A sector of a circle having a radius of 21 feet, contains 240°. Find the area in square feet.

5. The area of a 45° sector is 180 square feet. Find the radius of the circle.

6. Find the area of a sector of a circle, if the radius is 32 feet, and the part of the circumference 80 feet.

7 A sector of a circle is bounded by 120 feet of circumference, and two radii of 120 feet each. Find the area.

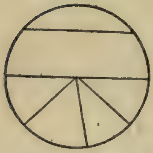
8. Find the number of degrees in the sector of the circle mentioned in question No. 6.

9. How many degrees are there in the sector mentioned in problem No. 7?

10. How many acres are there in a sector bounded by two radii 30 rods long and 45 rods of circumference?

11. The sector of a circle, 42 feet in diameter, contains 462 square feet. How many degrees are there between the radii?

Parts of Circles: The Segment.



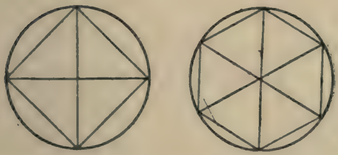
A segment of a circle is a part cut off by a line that does not pass through the centre. It may be either less or greater than a semicircle. In the annexed diagram what is above the line H K is a segment less than a semicircle, and the remainder of the circle below the line H K is a segment greater than a semicircle.

To measure a segment less than a semicircle, take the segment H G K, join the ends of the chord H and K with the centre O, and we have a sector. The sector contains the segment and a triangle. Find the area of the sector and the triangle, and subtract the triangle area from the sector area, and the segment area remains. To find the area of the segment larger than a semicircle, notice that the large segment is composed of *the triangle we deducted from the sector and the remainder of the circle*, add therefore the triangle to the portion that remained after the sector was taken away, and the result is the large segment;

or simply find the area of the small segment, and subtract from the area of the circle.

As the foregoing are only applications of the rules already given, it is not deemed necessary to give exercises.

A Square and Hexagon Inscribed in a Circle.



To find the side of a square inscribed in a circle. It will be observed from the figure annexed, that the diameter of the circle form~~s~~ the diagonal of the square, and it is the hypothenuse for two equal right angled triangles.

A hexagon may be inscribed in a square, and measured as per rule on page 39. The area of the six segments may be found by subtracting the area of the hexagon from the circle.

Rule. *Square the hypothenuse, divide by two, and extract the square root.*

EXAMPLE.

Find the side of a square inscribed in a circle 50 feet in diameter.

$$\frac{50\times 50}{2} = \sqrt{1250} = 35{\cdot}35 \text{ feet. Ans.}$$

EXERCISES.

1. How large a stick of square timber can be hewn from a log 20 inches in diameter at the small end?

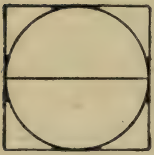
2. How large a stick of square timber can be sawn from a log 32 inches in diameter at the small end?

Solve the above exercises as indicated in the example.

Note. A short rule is sometimes given for finding the side of an inscribed square. "Multiply the diameter by .7071068." The author advises the student to work by general rules, rather than to attempt memorizing such decimal quantities.

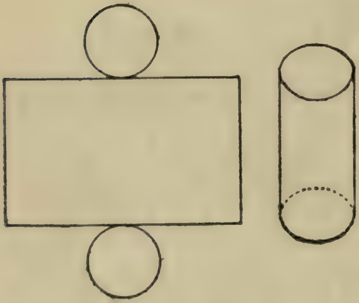
3. The area of a circle is 98.56 rods. A regular hexagon is inscribed in it. Find the difference between the area of the hexagon and that of circle.

A Circle Inscribed in a Square.



It will be observed by the accompanying diagram that the side of the square is equal to the diameter of the circle, and conversely the diameter of the circle is always equal to the side of the square in which it is inscribed. There is therefore, scarcely any exercise necessary on this point. It might be noticed also that the area of the corners remaining can be found by subtracting the area of the circle from the area of the square.

To Find the Area of a Cylinder.



This point may perhaps be best treated in connection with circles. The area consists of the areas of two ends and the area of surface in the form of a rectangle, (if cut and spread out,) the length being equal to the length of the cylinder, and the breadth to its circumference.

EXAMPLE.

Find the area of the outside of a cistern 7 feet in diameter and 8 feet long.

Area of two ends = $\frac{7}{2} \times \frac{7}{2} \times \frac{22 \times 2}{7} = 77$ square feet.

Area of outside = $\frac{7 \times 22}{7} = 22$ ft. circumference.

$22 \times 8 = 176$ square feet.

$176 + 77 = 253$ square feet. Ans.

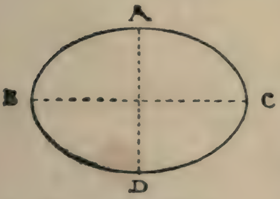
EXERCISES.

1. Find the area of a cylindrical cistern 10 feet diameter and 10 feet deep.

2. Find the area of the inside, (no ends,) of a conduit pipe, 4 feet in diameter and 2½ miles long.

3. Find the area of the inside of the City of Toronto water conduit from beyond Hanlan's Island, $\frac{7}{10}$ of a mile long, and 6 feet in diameter. (No ends.)

The Ellipse—To Find the Area.



An Ellipse is a regular figure, and easily drawn and measured. The line A B we will call the long diameter, and the half of it the long radius ; the line O E the short diameter, and half of it the short radius.

To draw an ellipse, draw the long and short diameters at right angles to each other the exact length required.

Measure the length of the long radius from the end of the short diameter O to where it would strike the long diameter at C and C on each side. Now stick pins at C, C and O. Tie a piece of string, that will not stretch, tightly around the three pins, then remove the pin at O, and with a lead pencil inside the string, pressing outwards against it, draw the ellipse, the string around the pins at C and C will control the direction.

To find the area, the rule is practically the same as the one for finding the area of a circle, the difference being that we have a long and a short radius.

RULE. *Multiply together the long and short radii, and multiply by* $\frac{22}{7}$ ($3\frac{1}{7}$).

NOTE. To find the area of an **elliptical ring,** find the areas of the larger and smaller ellipses, and find their difference, which is the area of the ring.

EXAMPLE.

Find the area of an ellipse, the diameters being 30 and 40 feet respectively.

Diameters being 30 and 40, the radii will be 15 and 20.

$$\frac{15 \times 20 \times 22}{7} = 942\tfrac{6}{7} \text{ square ft.}$$

EXERCISES.

1. Find the area of a fish pond in the form of an ellipse, 154 feet long and 60 feet wide.

2. Find the area, in acres, of an elliptical field, 84 rods long and 40 rods wide.

3. Find the area of an elliptical race course, the diameters of the larger ellipse being 280 and 140 yards, and of the smaller ellipse 266 and 126 yards.

The Circumference of an Ellipse.

The figure is akin to the circle, and so is the rule, except that two diameters must be used, and half the sum taken.

RULE. *Add together the long and short diameters: divide by 2 and Multiply by* $\frac{22}{7}$ $(3\frac{1}{7})$.

EXAMPLE.

Find the circumference of an ellipse, the diameters of which are 43 and 27 feet respectively.

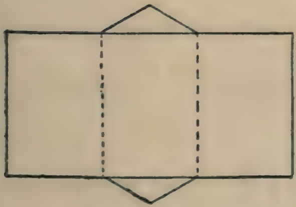
$$\frac{43+27}{2}=35 \qquad \frac{35\times 22}{7}=110 \text{ feet.}$$

EXERCISES.

1. Find the circumference of an ellipse having its diameters 18 and 24 feet respectively.

2. Find the circumference of an ellipse having its diameters 48 and 71 feet respectively.

The Area of a Prism.



A prism is a solid, whose ends are plane figures, equal, parallel and similar. Its sides are always parallelograms. The shape of the base determines the name. If the base is triangular we have a triangular prism. The method of measuring the area of a triangular prism is illustrated in the accompanying diagram. The sides opened out can be measured as one parallelogram, and the ends as two triangles.

RULE. ***Find the area of the ends and multiply the length of the prism by the distance around it, and add together.***

EXAMPLE.

A triangular prism is 6 feet high, the sides are respectively 3, 4 and 5 feet.

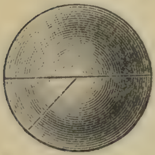
Area of ends = 6 sq. feet, area of 2 ends = 12 sq. feet.

The distance around the prism is 3 + 4 + 5 ft. = 12 sq. ft. × 6 = 72, area = 12 + 72 sq. ft. = 84.

EXERCISES.

1. Find the area of a triangular prism the ends of which are 4, 5 and 6 feet, and the height 10 feet.
2. Find the area of parallelopipedon 6 × 8 × 10 feet.
3. Find the area of a hexagonal prism 12 feet long, sides 6 feet.

The Area of a Sphere.



RULE. *Square the diameter and multiply by* $\frac{22}{7}$ ($3\frac{1}{7}$) or 3.1416.

EXAMPLE.

Find the area of a sphere, the diameter of which is 28 in.

$$\frac{28 \times 28 \times 22}{7} = 2464 \text{ square inches.}$$

EXERCISES.

1. A football is 10 inches in diameter. Find the area of the surface.

2. The earth is 8000 miles in diameter. Find the surface in square miles.

3. Find the surface of a sphere 84 feet in circumference.

4. The circumference of a sphere is 88 inches. Find the area of the surface.

The Area of the Curved Surface of Cones.

Cone.

Suppose we take a circular tent and open up one of the seams and spread it out on the ground. We will find that the tent is a sector of a circle. The higher the pitch of the tent, the smaller the angle between the radii of the sector. Draw a circle on paper, and cut it out. Cut out a sector from it, and try the experiment of setting up a tent. The slant height of the tent corresponds with the radius of the circle. The rule, therefore, for finding the area of the curved surface of a cone is the same as for finding the area of a sector, page 50, *viz.*,—

RULE.—*Multiply the slant height of the cone by the circumference, and divide by two.*

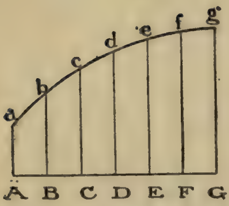
EXERCISES.

1. Find the area of a cone, the slant height of which is 36 feet and the circumference 80 feet.

2. A tent is 42 feet in circumference, and 15 feet slant height. How many yards of canvas are required to make it, (no allowance for waste).

3. How many yards of canvas are in a tent 42 feet in diameter, and 30 feet slant height?

Simpson's Rule.



The following rule, generally known as Simpson's rule, sometimes as the rule of Equi-distant Ordinates, is useful in finding the areas of irregular figures, approximately. The greater the number of ordinates, the nearer the approximation. Take for an example, an area having one side curved, the others straight lines at right angles to the base line. Divide the line A G, into an even number of equal parts at B, C, D, E, F.

Draw straight lines through the points at right angles to A G, meeting the curved line at b, c, d, e, f. These lines are called ordinates.

RULE. *Add together the first and last ordinates, twice the sum of all the other odd ordinates and four times the sum of the even ordinates. Multiply the result by one-third of the distance between the ordinates.*

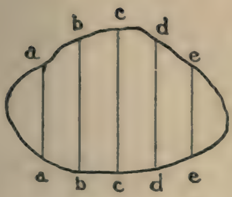
Let the following be the ordinates in their order, beginning with A a. 4, 5, 6, 7, 8, 9, 10 feet, and the distance between the ordinates $1\frac{1}{2}$ feet:—

$4 + 10 = \quad 14$
$6 + 8 = 14 \times 2 = \quad 28$
$5 + 7 + 9 = 21 \times 4 = \quad 84$
$\quad 126$

$$\frac{126 \times 1\frac{1}{2}}{3} = 63 \text{ sq. feet.}$$

EXERCISES.

1. The ordinates in a figure are 4, 14, 36, 76, 140 feet and the common distance one foot. Find the area.

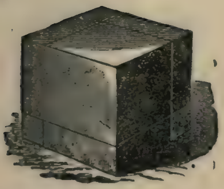


Simpson's rule may be applied to almost any irregular figure. Take the annexed irregular oval for an example. The first and last ordinates will of course be 0.

2. Suppose the ordinates are 0, 20, 32, 36, 32, 20, 0, and the common distance 2 feet. Find the area.

3. The ordinates of a certain irregular field are 0, 12, 16, 18, 20, 24, 22, 18, 16, 15, 0 rods, and the common distance between them $2\frac{1}{4}$ rods. Find the area in acres, &c.

Mensuration of Solids.



A solid has length, breadth, and thickness or height.

Volume, solidity, or solid contents of any body is the number of cubic inches, feet, yards, &c., it contains,

It will be remembered that to find area, *two* dimensions were multiplied together. To find volume, *three* dimensions must be multiplied together—length, breadth, and thickness.

The connection between the tables of long, square, and cubic measure, should also be noticed :—

12 in. = 1 foot. 12 × 12 = 144 sq. inches = 1 sq. foot.
12 × 12 × 12 = 1728 cubic inches = 1 cubic foot.
3 feet = 1 yard. 3 × 3 = 9 sq. feet = 1 sq. yd.
3 × 3 × 3 = 27 cubic feet = 1 cubic yard.

The following are a few equivalents in weights and measures :—

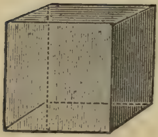
1 cubic foot of water weighs 62½ lbs.

1 imperial bushel contains 2218.19 cubic inches.

Where absolute accuracy is not required, 25 quarts may be counted a cubic foot.

A short rule for reducing cubic feet to bushels is—Multiply the cubic feet by 100 and divide by 128. This rule is not a half a bushel astray per thousand bushels.

The Cube.



To find the volume of a cube.

Rule. *Multiply together the length, breadth, and height.* (*In a cube these are equal.*)

EXAMPLE.

Find the volume of a cubical block of wood, each side being 15 inches. 15 × 15 × 15 = 3375 cubic inches.

EXERCISES.

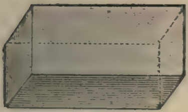
1. Find the volume of a cube, the side of which is 8 ft.
2. Find the volume of a cube 5 feet square.
3. How many cubic yards in a cube 15 feet square?
4. Find the length of the side of a cube containing 13824 cubic feet. 24 ft

Note. Extract the cube root of the volume.

5. A farmer borrows a piece of hay 1 yard long, 1 yard wide, and 1 yard thick from his neighbor, and pays him

back in pieces half a yard long, half a yard wide, and half a yard thick. How many such pieces should he return him for what he borrowed?— = 8 pieces

The Parallelopipedon.



To find the volume of a parallelopipedon.

RULE. *Multiply together the length, breadth, and thickness (or height.)*

EXAMPLE.

Find the volume of a piece of marble 6 feet long, 3 feet wide, and 2 feet thick 6 × 3 × 2 = 36 cubic feet.

If the volume and two of the dimensions of a parallelopipedon are given, the third dimension may be found by dividing the volume by the product of the given dimensions.

EXAMPLE.

The volume of a parallelopipedon is 180 cubic feet, the breadth is 5 feet, and the thickness 4 feet. Find the length.

$\frac{180}{5 \times 4} = 9$ feet long.

EXERCISES.

1. Find the cubic yards of earth excavated from a cellar 30 feet long, 24 feet wide, and 6 feet deep.

2. Find the cost of excavating a cellar 45 feet long, 36 feet wide, and 5 feet deep, at 20 cents per cubic yard.

3. Find the volume, in cubic feet, of a stick of timber 60 inches long, 21 inches wide, and 15 inches thick.

4. Find the cubic feet in a parallelopipedon, 7 feet 9 in. long, 4 feet 6 inches wide, and 2 feet 3 inches high.

5. How many bricks, each 8 inches long, 5¼ inches wide, and 2¼ inches thick, are there in a pile 3 feet 6 inches long, 3 feet wide, and 2 feet high?

6. Prove that a 6-inch cube is equal in volume to the sum of the volume of a 3 inch, a 4 inch, and a 5 inch cube.

7. A swimming bath is 24 feet 8 inches long, by 12 feet 9 inches wide. How many gallons of water are required to fill it 4 feet deep? (25 quarts to cubic foot.) 277. 274

8. 144 cubic yards of earth were excavated from a cellar 24 × 36 feet. Find its depth.

9. A cubical block of stone weighing 23040 lbs., weighs 15 lbs., to 72 cubic inches. Find the size of the block.

Diagonal of Cube or Parallelopipedon.

To find the diagonal of a cube or a parallelopipedon.

RULE. *Find first the diagonal (hypothenuse) of the base by squaring the sides, adding together and extracting the square root. Next use this diagonal as a base of a triangle and the height as the other side, and repeat the operation of finding the diagonal (hypothenuse). The last found is the required diagonal of the cubical figure.*

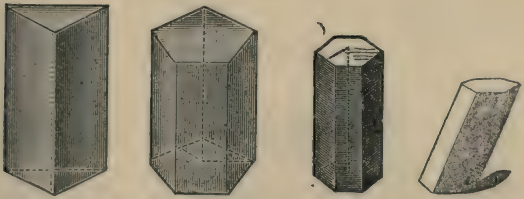
EXAMPLE.

A parallelopipedon is 40 feet long, 30 feet wide, and 20 feet thick. Find its diagonal.

First find diagonal of base $\sqrt{(30^2 + 40^2)} = 50$ feet the diagonal of the base, then using this diagonal as the base and the height, 20 as the height, the diagonal of the cube is found thus $\sqrt{(50^2 + 20^2)} = 53.85$ feet, the diagonal required.

1. The north and south sides of a room are 30 ft. long; the east and west sides are 24 ft. The height is 16 ft. Find the distance from the north-east top corner to the south-~~east~~ west bottom corner.

The Prism.



A prism is a solid the ends of which are plane figures, equal parallel and similar. Its sides are always parallelograms. The shape of the base determines the name. If the base is triangular we have a triangular prism.

RULE. *Multiply the area of the base by the perpendicular height.*

EXAMPLE.

Find the volume of a triangular prism 70 feet high, the sides of the base being 30, 40, and 50 feet.

Area of base 600 feet. $600 \times 70 = 42000$ cubic feet.

EXERCISES.

1. Find the cubical contents of a square prism 8 feet high, the area of the base being 6 square feet.

2. Find the volume of a hexagonal prism 15 inches high, if the area of the base is 12½ square inches.

3. Find the volume, in cubic feet and inches, of a hexagonal prism having a base of 35 square feet 123 square inches area, and a height 5 feet 5 inches.

4. The volume of a pentagonal prism is 28 cubic feet and 500 cubic inches. The area of the base is 7 square feet and 103 square inches. Find the height.

5. A triangular prism measures 12 inches on every edge. Find the volume.

The Cylinder.

The cylinder is a solid, having circular ends. As the figure is like a prism, except that the base or end is circular, it might be called a circular prism.

To find the volume of a Cylinder.

RULE. *Multiply the area of the end by the length or height.*

EXAMPLE.

A cylindrical cistern is 7 feet in diameter and 10 feet deep. Find its volume.

$\frac{7}{2} \times \frac{7}{2} \times \frac{22}{7} = \frac{77}{2} = 38\frac{1}{2}$ square feet area of base. $38\frac{1}{2} \times 10 = 385$ cubic feet = volume.

EXERCISES.

1. A length of 12 inch water pipe is 14 feet long. How many cubic feet of water will it contain?

2. Find the volume of water in a cylindrical well 4 feet in diameter, depth of water 35 feet.

3. The length of a round log is 18 feet, and its circumference 96 inches. Find its volume in cubic feet.

4. Suppose a 25 cent piece is $\frac{7}{8}$ of an inch in diameter and $\frac{1}{16}$ of an inch thick. If $25,000 in quarter dollars were melted into a cube, find its size. 15.54 inches

5. How many cubic feet in a log 12 feet long and 18 inches in diameter?

6. A water pipe is one inch thick and the inner diameter is ½ inch. How many cubic inches of metal are there in 20 ft. of pipe.

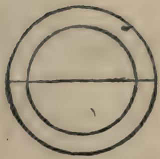
7. What length of wire could be made from 1 cubic foot of brass, if the wire is $\frac{1}{20}$ inch in diameter. 13 miles 4669 $\frac{1}{11}$

8. A piece of brass contains 6000 cubic inches. Suppose it were made into wire $\frac{1}{20}$ inch thick, find how long the wire would be. 254545 $\frac{5}{11}$ ft.

9. How many cubic feet are contained in the wall of a round tower, the height of which is 60 feet, the thickness of wall 2½ feet. The tower covers a piece of ground 20 feet in diameter.

10. How many cubic inches of metal are contained in a ¾ inch water pipe, the thickness of the metal being ½ inch, and the length of the pipe 10 feet.

The Ring.



It will be evident to the student that the same rule that applies to the cylinder will apply to a ring, if made from square material, viz., multiply the area of the base by the thickness. For area of the base see page 49.

A ring made from round material, however, may present some little difficulty.

RULE. *Multiply the area of a cross section of the ring by the length of the ring.*

The cross section would be the end of the metal, or other material, if the ring were cut through.

The length of the ring would be the distance around at the middle of the metal substance of which it is composed. Thus if the diameter of the outside were 20 inches, and of the inside 16 inches, the diameter for finding the length of the ring would be found thus :—$\frac{20 + 16}{2} = 18$

EXAMPLE.

If the inside and outside diameters of a ring made from round iron are 16 and 20, find the volume. It is evident that the diameter of the iron is two inches, and the radius one inch $\frac{1 \times 1 \times 22}{7} = 3\frac{1}{7}$ square inches area of cross section.

$\frac{18 \times 22}{7} = \frac{396}{7} = 56\frac{4}{7}$ inches = length of ring.

$\therefore 56\frac{4}{7} \times 3\frac{1}{7} = 177\frac{39}{49}$. Ans.

EXERCISES.

1. A cylindrical ring is 20 inches long, and $1\frac{3}{4}$ inches diameter cross section of metal. Find the volume.

2. A cylindrical ring is 7 inches thick. The inner diameter is 20 inches. Find the volume.

3. A ring 35 inches in diameter is made from iron $1\frac{1}{2}$ inches square. Find the volume.

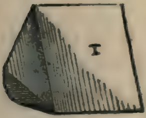
4. A hoop on a kiln 36 feet in diameter is made from iron 1 × 4 inches. Find its volume.

Portions of Cylinders.

Any portion of a cylinder having a base, a segment, or sector, may be measured by the same rule, *viz.* :—Multiply the area of the base by the perpendicular height.

1. Find the volume of a 60 degree sector of a cylinder 14 inches in diameter and 3 feet long.

The Wedge.



The wedge is simply a triangular prism turned on its side, and may be measured as such.

If, however, the sides of the base and the perpendicular height be given, the contents are easily found.

Note. By inspection it will be observed that the wedge is just half of a parallelopipedon or a cube. Hence the following rule :—

Rule. *Multiply the area of the base by the perpendicular height, and divide by two.*

EXAMPLE.

The base of a wedge is 4 × 6 inches, and the perpendicular height 10 inches. Find the volume.

$$\frac{4\times6\times10}{2} = 120 \text{ cubic inches} = \text{volume.}$$

EXERCISES.

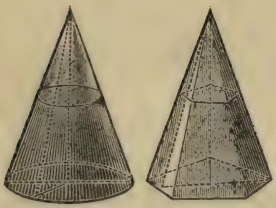
1. The base of a wedge is a foot square, and the perpendicular height three feet. Find the volume.

2. The base of a wedge is 8 × 12 inches, and the volume 210 cubic inches. Find the perpendicular.

3. The base of a wedge is 3 × 3 inches, and the slant height 8 inches. Find the volume.

4. Find the volume of a wedge 5 × 2 inches base, and 10 inches slant height.

Pyramids and Cones.



A pyramid is a solid having a plane figure for its base, and triangular sides that meet in a point called the vertex.

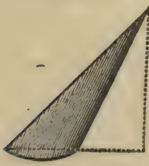
Pyramids are named according to the shape of their bases. A triangular pyramid has a triangular base. A quadrilateral pyramid has a quadrilateral base, &c.

A cone is a pyramid with a circular base. A pile of sand or grain, or any such substance is a cone.

The wedge, in the preceding chapter, sloped from two sides, the volume was found by multiplying the area of the base by the height and dividing by two. The pyramid slopes to a point from all its sides, therefore the divisor must be greater than two.

To find the volume of a pyramid or cone.

RULE. *Multiply the area of the base by the perpendicular height, and divide by three.*



NOTE. It will be observed the words "perpendicular height" are used. From the accompanying diagram it will be seen how this is found. The proof will be evident to any student who has studied the first book of Euclid.

EXAMPLE.

Find the volume of a pyramid 15 feet high, having a base 6 feet square $\frac{6 \times 6 \times 15}{3} = 180$ cubic feet.

Find the volume of a cone 14 feet in diameter, and 24 feet high.

Radius = 7 feet $\frac{7 \times 7 \times 22 \times 24}{7 \times 3} = 1232$ cubic feet.

EXERCISES.

1. A cone is 16 feet in diameter and 15 feet high. Find the volume in cubic feet.

2. The base of a pyramid is 12 feet square, height 20 feet. Find the volume.

3. The sides of the base of a triangular prism are 4, 5, and 7 feet, and the height 6 feet. Find the volume.

4. The base of a rectangular pyramid is 15 × 20 feet, and the height is 18 feet. Find the volume.

5. The base of a pyramid has an area of 20 square feet, 120 square inches, and the height is 5 feet 8 inches. Find the volume.

6. A ship's mast 100 feet high, is 3 feet in diameter at the base, and pointed at the top. Find the volume in cubic feet.

7. A side of the base of the hexagonal pyramid is 6 feet, the perpendicular height is 15 feet. Find the volume.

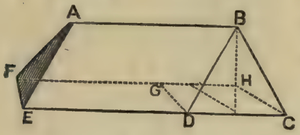
8. A pile of grain is 18 feet in diameter, and 8 feet high. How many bushels does it contain? (2218.19 cubic inches to the bushel).

9. A circular reservoir is 30 feet in diameter, 3 feet deep at the edge, and 8 feet in the centre. How many gallons will it contain? (277.274 cubic inches to the gallon).

10. Find the volume of a pyramid 300 feet square at the base, the slant height from the middle of a side to the vertex being 200 feet.

11. The great pyramid of Egypt was 481 feet high when completed, and its base was a square, 764 feet in length. Find its volume in cubic yards.

12. The spire of a church is a pyramid on a regular hexagonal base. Each side of the base is 10 feet, and the height 60 feet. The spire is built of brick, hollow, the same shape inside as outside. The base on each side is 9 feet., and the height inside 54 feet. Find the volume of the brick work in cubic feet.



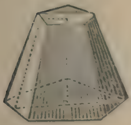
The following diagram shows a very common figure, and will illustrate a very nice application of the rules for measuring the volume of a wedge and pyramid. If this figure were sawn down through perpendicularly at A and B, the middle portion would be a wedge, and the two ends brought together would form a pyramid—B, D, G, H, C.

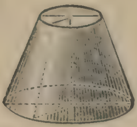
EXERCISES.

13. Suppose the base of the figure above is 20 feet long, and 8 feet wide, and the perpendicular height 18 feet. Find the volume.

14. Find the volume of vessel similar to above figure turned upside down, size 6 × 8 by 10 feet deep.

The Frustum of the Pyramid and Cone.





A frustum of a pyramid or cone is simply that part of a pyramid or cone that remains after the top has been cut off parallel with the base.

To find the volume of a frustum of a pyramid or cone.

RULE. *Add together the areas of the ends and the square root of the product of the areas of the ends. Multiply by the perpendicular height and divide by three.*

EXAMPLE.

Find the volume of the frustum of a pyramid 12 feet square at the base, 3 feet square at the top, and 15 feet high.

Area of the small end $3 \times 3 = 9$
" " large end $12 \times 12 = 144$
Square root of their product $= \sqrt{9 \times 144} = 36$
189

$\frac{189 \times 15}{3} = 945$ cubic feet. = volume.

EXERCISES.

1. Find the volume of a frustum of a pyramid 10 × 10 ft. at base, 4 × 4 feet at top, and 15 feet high.

2. Find the volume of a frustum of a pyramid 10 × 10 ft. at base, 3 × 3 feet at top, and 12 feet high.

3. Find the volume of a frustum of a cone 14 feet radius, at base, 7 feet radius at top, and 30 feet high.

4. Find in cubic feet the volume of a frustum of a cone 12 feet high, the area of the large end being 36 square feet, and the small end 16 square feet.

5. The mast of a ship is 66 inches in circumference at the base, 44 inches at top and 45 feet high. Find the volume in cubic feet.

6. How many gallons will a circular tank contain that is 10 feet diameter at bottom, 8 feet at top, and 6 feet deep. (Gallon = 277.274 cubic inches).

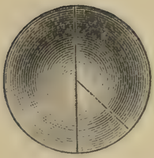
7. Find the volume of the frustum of a pyramid, the base of which is 18 × 24 feet, the top 6 × 8 feet, and the height 15 feet.

8. A cistern 5 × 12 feet at the bottom, 8 × 15 feet at top, and 10 feet deep, will contain how many gallons?

9. A frustum of a triangular pyramid is 20 feet high, the sides of the base are 6, 8 and 10 feet, and the sides of the top 3, 4 and 5 feet respectively. Find the volume.

10. A circular stack of new hay is 44 feet circumference at the base, 66 feet circumference at the largest part, 15 ft. from the ground. The top part is 30 feet high above this. Find the number of tons in it, if 24 cubic yards make a ton.

The Sphere.



A sphere is a round solid body.

To find the volume of a sphere.

RULE. *Cube the diameter. Multiply by* $\frac{22}{7}$, *(or* 3.1416 *more accurately) and divide by six.*

NOTE 1. The rule for finding the volume of a sphere may be stated in many ways. One very common way is, volume = (radius)$^3 \times \frac{4}{3} \times \frac{22}{7}$

NOTE 2. The volume of a spherical shell will be found by subtracting the volume of the inside from the volume of the shell, or by

finding the difference of the cubes of the radii, multiplying by $\frac{22}{7}$ and dividing by 6.

EXAMPLE.

Find the volume of a sphere 4 feet in diameter.

$$\frac{4 \times 4 \times 4 \times 22}{7 \times 6} = 33\tfrac{11}{21} \text{ cubic feet} = \text{volume.}$$

EXERCISES.

1. Find the volume of a football 1 foot in diameter.

2. If the diameter of the earth is 8000 miles, find its volume in cubic yards.

3. Find the volume of a globe 3 feet in diameter.

4. Find the volume of a sphere 15 inches in diameter.

5. Find the volume of a sphere 16 inches in diameter.

6. Find the weight of a cannon ball 5 inches in diameter, made from metal weighing 700 lbs. per cubic foot.

7. A cannon ball, 9 inches in diameter, is made from metal weighing 840 lbs. to the cubic foot. Find its weight. 185.62

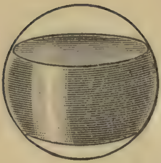
8. Find the weight of a spherical shell 14 inches in diameter, 2 inches thick, shell made from metal weighing 800 lbs. per cubic foot, and filled with explosive weighing 200 lbs. per cubic foot. $483\frac{314}{367}$

9. A spherical balloon is filled with slat gas that has a lifting power of 21 ounces to the cubic foot. What size should it be so as to carry two men and fittings, weighing in all 600 lbs ?

10. Find the lifting power of a spherical balloon 50 feet in diameter, filled with gas that has a lifting power of 22 oz. per cubic foot.

11. A spherical balloon is filled with gas that has a lifting power of $17\frac{1}{2}$ oz. per cubic foot, and carries a load of 1000 lbs. Find its diameter 12.16

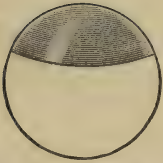
The Zone of a Sphere.



To find the volume of a zone of a sphere.

RULE. *Add together three times the sum of the squares of the radii of the two ends and the square of the height. Multiply by the height and by $\frac{22}{7}$, and divide by 6.*

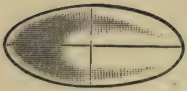
The Segment of a Sphere.



To find the volume of the segment of a sphere.

RULE. *Add together three times the square of the radius of the base and the square of the height. Multiply by the height and by $\frac{22}{7}$ and divide by 6.*

The Oblate Spheroid.

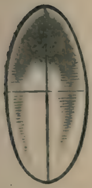


An Oblate is a sphere partly flattened. A curling stone is a common example of an oblate spheroid. The curling stone, however, is flattened at the top and bottom, and a spheroid is not.

To find the volume of an oblate spheroid.

RULE. *Multiply the square of the short diameter by the long diameter. Multiply by $\frac{22}{7}$, and divide by six.*

Prolate Spheroid.



A prolate spheroid is like a ball elongated, or like a bird's egg, except that the egg is larger at one end than at the cther, and the prolate spheroid is the same at both ends.

To find the volume of a prolate spheroid.

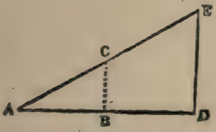
RULE. *Multiply the square of the short diameter by the long diameter. Multiply by* $\frac{22}{7}$ *and divide by six.*

Similar Figures.

In order that figures may be similar, they must be proportionate in their dimensions, and those containing angles must have their corresponding angles equal.

All circles are similar. All squares are similar; So are pentagons, hexagons, octagons. Triangles, and parallelograms, are similar, if their corresponding angles are equal.

Similar triangles can be used in determining distance of objects out of reach, by the use of simple articles, such as a square and measuring pole or tape line, and a few pegs or pickets.



Similar triangles may be formed by drawing a line through any triangle parallel to a side. Take the triangle A E D, draw through it the line B C.

The small triangle is similar to the whole triangle because the angle at C is equal to the angle at E. The angle at B to the one at D, and the angle at A common to both.

RULE. *The corresponding or homologous sides of similar triangles are proportionate.*

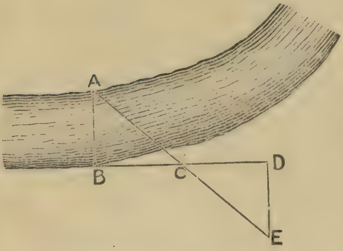
To find the side D E of the large triangle, the side A D being given, and the two corresponding sides of the smaller one given.

EXAMPLE.

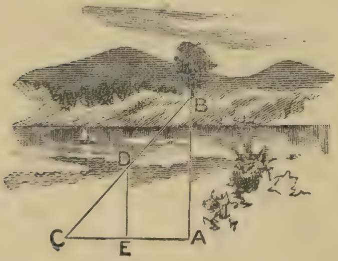
If the side A B is 8 feet, and B C 6 feet, and the side A D 40 feet.

Let x = the side D E, then $8 : 40 :: 6 : x$

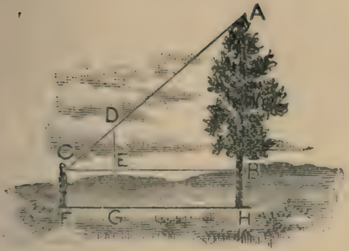
$\therefore x = 30$ feet = D E.



The annexed figure illustrates a method of finding the breadth of a river by laying out similar triangles with a few pegs and a carpenter's square. It will always be found convenient to lay out right-angled triangles in such work.



The subjoined illustrations show a method of measuring the distance of an inaccessible object—a tree beyond a river, and the height of a standing tree. The latter may also be conveniently measured sometimes by setting



up a stake or post and measuring the shadow cast by each. Then knowing the height of the post, the height of the tree may be found by an application of the rule regarding similar figures.

Areas of Similar Figures.

RULE. *The areas of similar figures are in proportion as the squares of corresponding or homologous dimensions.*

EXAMPLE.

The area of a circle, 14 feet in diameter, is 154 square feet. What would be the area of a circle 25 feet in diameter ?. The corresponding dimensions given, are diameters—

$14^2 : 154 :: 25^2 : x \therefore 196 : 154 :: 625 : x, x = 491\frac{1}{14}$ = area of circle.

EXERCISES.

1. A tract of land in the form of a right-angled triangle is owned jointly and equally by A B and C. The sides containing the right angle are 15 and 20 miles respectively. Two lines are surveyed at right angles to the 20 mile side, and parallel to the 15 mile side. A has the small triangle, B the middle portion, and C the portion lying along the 15 mile side. Find the frontage each man has on the 20 mile boundary.

2. A carriage wheel, 3 feet in diameter, is set in motion on the ground at an inclination of one inch from the perpendicular. Find the diameter of the circle it will traverse.

Similar Solids.

One solid body is similar to another if they are alike in **form**, though they differ in size. In other words, when one is the model of the other.

All cubes are similar solids. All spheres are similar. Other solids may be similar. The following rule will hold good in all similar solids.

RULE. *The volumes of similar solids are in proportion to the cubes of their similar dimensions.*

EXAMPLE.

If a sphere $3\frac{1}{2}$ inches in diameter weighs 6 lbs., find the weight of a sphere $5\frac{1}{4}$ inches in diamerer.

$$(3\tfrac{1}{2})^3 : 6 :: (5\tfrac{1}{4})^3 : x,\ x = 20\tfrac{1}{4}$$

The model of a machine weighs 100 lbs. Find the weight of a machine made from the same material five times its linear dimensions.

$1^3 : 100 :: 5^3\ x\ x = 12500$ lbs., the weight of the machine.

Irregular Solids.

A simple plan to find the volume of an irregular body, is to fill a vessel, sufficiently large, with water. Immerse the solid in the water and measure the water that ran over the vessel.

An application of Simpson's rule is sometimes used in estimating earth excavations, &c.

RULE. *Divide the solid into any even number of portions, (by length). Ascertain the areas of the several cross sections at the points of division perpendicular to the length of the solid. Add together the first area and the last area, and twice the sum of the other odd areas, and four times the sum of all the even areas, and multiply the sum by one-third of the common distance between them.*

COMMON MEASUREMENTS.

The following pages contain many every day problems that are so simple that it has not been deemed necessary to add much explanation nor to work out many problems, almost all the work being applications of the rules already learned, if the student has worked from the beginning. A great many of the examples may be worked by a person who has only learned the "four simple rules in arithmetic."

Wood Measure.

A cord of wood contains 128 cubic feet.

A standard cord pile is 8 feet long, 4 feet wide and 4 feet high—$8 \times 4 \times 4 = 128$ cubic feet.

RULE. *Find the volume of the pile in cubic feet by multiplying together the length, breadth and height, and divide by 128.*

EXAMPLE.

How many cords of wood are there in a pile of 4-foot wood, 40 feet long, 6 feet high.

$$\frac{40 \times 4 \times 6}{128} = 7\tfrac{1}{2} \text{ Cords.}$$

EXERCISES.

1. How many cords of wood are there in a pile 20 feet long, 4 feet wide and 4 feet high?

2. How many cords in a pile 60 feet long, 4 feet wide and 4 feet high?

3. How many cords in a pile 64 feet long, 4 feet wide, and 7 feet high?

4. How many cords in a pile 119 feet long, $7\frac{3}{4}$ feet high, 18 feet wide?

5. How many cords in a pile 77 feet long, 4 feet wide, 9 feet high?

6. Find the cost of the wood in exercise No. 4, at $2.75 per cord.

7. How high must 4-foot wood be piled on a sleigh rack, 11 feet long, to make $1\frac{1}{2}$ cords?

8. My wood-shed is 20 feet long by 16 feet wide. How high must I pile wood in it in order to put 18 cords in it?

9. How high should wood be piled between two stakes 6 feet 4 inches apart so as to make a cord?

10. A sleigh rack is 9 feet 6 inches long. How high must wood be piled in it to make a cord?

11. If wood is piled 7 feet 4 inches high in a shed 30 feet long, 22 feet wide, how many cords will it contain?

12. How many cords in a pile of tan bark 300 feet long, 8 feet wide, 5 feet high? $93\frac{3}{4}$

13. A wood rack is 11 feet 6 inches long. How many loads of 4-foot wood, 4 feet high on such rack will be required to make a pile 46 feet long, 40 feet wide, 6 feet high?

14. Find the value of the wood in exercise No. 13, at $3.15 per cord.

15. Find the value of a load of wood on a rack 10 feet 8 inches long, 4 feet wide and 4 feet 4 inches high, at $3.25 per cord.

16. What length of a pile of 30-inch wood, 4 feet high, would be equivalent to a cord?

17. What length of a pile of 18-inch wood, 4 feet high, would be equal to a cord?

18. What height should a pile of 20-inch wood, 10 feet long, be to contain a cord?

19. A single pile of 20-inch wood is 5 feet high, 30 feet long. How many cords does it contain?

20. A pile of wood, 12 feet long, 6 feet high, contains a cord. Find the length of the wood.

21. A flat car, 33 feet long, 8 feet wide, is loaded 8 feet high with wood. How many cords are on it?

22. Find the value of a load of 4-foot wood, 3 feet 9 inches high, on a rack 9 feet 8 inches long, at $2.75 per cord.

Lumber Measure.

Lumber is usually sold by the 1000 square feet. A foot of lumber is 1 foot square and an inch thick. If less than an inch in thickness it is counted an inch thick, that is, surface measure is counted. If more than an inch in thickness, the thickness is counted. A piece of board a foot square, 2 inches thick, would contain 2 feet of lumber; if 3 inches thick, 3 feet of lumber; if 1½ inches thick, 1½ feet, &c.

Lumber includes boards, scantlings, joists, planks, &c.

RULE. *Multiply the length by the breadth, for all lumber not more than one inch thick, to find the area. If thicker than one inch, multiply the area by the thickness in inches.*

NOTE. Although the exact mathematical answer is given in these lumber exercises, in practice the nearest even inches are counted in

breadth and the nearest even feet in length, and in fractions of feet the nearest even feet are taken in final results for finding cost.

EXERCISES.

1. How many feet of lumber in a board 12 feet long, 12 inches wide, 1 inch thick?

2. How many in the above board if 2 inches thick? if 3 inches thick? if $3\frac{1}{2}$ inches thick?

3. How many feet in the board in exercise No. 1, if $\frac{1}{2}$ an inch thick?

4. How many feet of lumber in 12 boards, 12 feet long, 5 inches wide, 1 inch thick?

5. How many feet in 90 pieces, 12 feet long, 2 × 4 inches?

6. How many feet of lumber is 100 pieces, 2 × 4 inches, 14 feet long.

7. How many feet in 12 pieces 4 × 4, 16 feet long.

8. How many feet in 24 half-inch boards, 10 inches wide, 15 feet long?

9. How many feet in the following bill? Also find the cost at $7.00 for all that is less than 18 feet long, and $8.00 for all that is 18 feet long and over.

200 boards,	12	feet long,	10	inches wide,	1	inch thick.
100 "	14	"	9	"	1	"
25 "	18	"	12	"	1	"
40 joists	20	"	10	"	2	"
180 scantlings	16	"	4	"	2	"

10. The following are the breadths of ten planks, each 14 feet long, 2 inches thick: 11, 12, 10, 12, 11, 9, 8, 6, 7, 10. How many feet do they contain?

11. How many feet are there in 30 boards, 1 inch thick, 10 feet long? The following are their widths in inches: 12, 13, 16, 10, 9, 13, 8, 7, 12, 6, 8, 4, 9, 12, 13, 13, 14, 15, 14, 15, 13, 9, 8, 9, 9, 9, 8, 7, 5, 3.

12. How many feet of lumber are required to lay the floor of a stable, 20 × 30 feet; the floor to be 2 inches thick?

13. How many feet of matched lumber are required to lay an inch floor in a house 24 × 36 feet, if one eighth of the surface is lost in matching?

14. How many scantlings are required for a partition 40 feet long, if they are placed at 16 inch centres?

15. How many feet of lumber are required for the sheeting of a building 22 feet long, the sheeting to project a foot over each end, the rafters being 12 feet long?

16. Find the cost of the lumber for a slatted walk, 160 feet long, 3 feet wide, the slats being 2 inches wide, 1½ inches thick, placed 1 inch apart, at $18.50 per 1000 feet, and three 4 × 4 scantlings, the entire length underneath, at $8.75 per 1000.

17. A sidewalk made of 2 inch plank is 220 rods long, 12 feet wide. There are 5 scantlings, 3 × 6 inches, underneath the entire length. Find the cost of the lumber at $11 00 per 1000.

18. How many feet of lumber are required to close in the sides and ends of a barn, 40 × 60 feet, 18 feet high, the gable ends being 15 feet high?

19. Find the cost of the following bill of lumber. Under 16 feet long at $7.50, and 16 and over at $8.00 per 1000.

Make out the bill neatly.

32 joists	2 × 10	20	ft. long.
32 "	2 × 8	20	"
64 rafters	2 × 6	15	"
220 scant's	2 × 4	14	"
125 "	2 × 4	12	"

200 boards	1 × 12	14	ft. long.
2 sticks	9 × 9	30	"
2 "	9 × 9	20	"
200 battens at 3 cents each.			

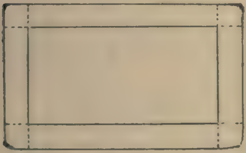
20. A piece of timber in the form of a wedge is 12 inches square at one end and sharp at the other, 14 feet long. How many feet of lumber are there in it?

21. A 3-inch plank, 20 feet long, is 12 inches wide at one end, and 20 inches at the other. How many feet of lumber are there in it?

22. Find the number of feet of lumber in the following bill, and the cost of same at $8.00 per 1000 feet when less than 20 feet long, and $10.00 for all that is 20 feet and over.

300 pieces	2 × 4,	16	ft. long.
200 "	2 × 4,	12	"
200 "	2 × 4,	10	"
100 "	2 × 10,	28	"
100 "	2 × 8,	28	"

400 pieces	2 × 2,	12	ft. long
300 "	1 × 12,	12	"
210 "	2 × 6,	18	"
2 "	6 × 8,	28	"
2 "	6 × 8,	36	"

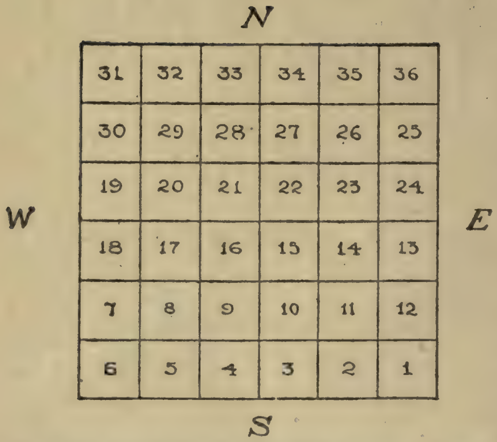


23. Find the cost of laying a 6-foot plank side walk around a block 420 × 630 feet, plank 2 inches thick at $15.00 per thousand; 4 string pieces, the entire length, 4 × 4 at $12.00 per thousand, spikes $6.75; labor $48.50.

Land Measure.

At present all government lands are laid out in Townships of 6 miles square. All the older settled parts of

Canada are laid out irregularly. Under the new system of survey, a Township contains 36 square miles. Each square mile is called a section and contains 640 acres. These are again sub-divided into half sections (320 acres), quarter sections (160 acres) &c. The townships are laid out and numbered like the accompanying diagram.



31	32	33	34	35	36
30	29	28	27	26	25
19	20	21	22	23	24
18	17	16	15	14	13
7	8	9	10	11	12
6	5	4	3	2	1

EXERCISES

1. What are the dimensions in rods of a quarter section of land?

2. Find the value of a Township of land in Manitoba, at 50 cents per acre.

3. If a quarter section of land is worth $240.00, what is the price per acre?

4. How many quarter sections of land are there in a tract 12 miles square?

5. Find the area in acres of a field 30 × 40 rods.

6. Find the area in acres of 5 miles of road, 66 feet wide.

7. How many acres are there in a field 40 × 60 rods?

Find the side of a square field equal to it in area.

8. A piece of land in the form of a sector of a circle has a radius of 42 rods, and an angle between the radii of 40°. Find the area.

9. A trapezium has its sides 40, 60, 70, 80, rods, the diagonal is 100 rods, forming triangles having their sides 60, 70, 100, and 80, 40, 100. Find the area.

10. A farm has a frontage of 30 rods on a concession, and contains 50 acres. Find its length.

11. A quarter acre lot is 210 feet in depth. Find its frontage on the street.

12. How many acres, etc., are there in a circular plot of ground 28 rods in diameter?

13. A circular field contains $5\frac{1}{2}$ acres. Find its diameter.

14. A hexagon is drawn in a circle having a radius of 42 rods. How much land is cut off as a segment of the circle by one of the sides of the hexagon? Find the number of acres in the hexagon.

Measurement of Lathing.

Laths are sold by the 1000. They are put up in bunches of 50.

A lath is 4 feet long and $1\frac{1}{2}$ inches wide. They are put on an average of $\frac{3}{8}$ of an inch apart.

Contractors usually estimate 1000 laths as covering 60 square yards, or 3 yards to the bunch. This makes sufficient allowance for waste. Good lath, carefully put on, will usually cover from 65 to 68 square yards. The contractors' rule of 60 yards to the 1000 laths is to be used in these exercises unless otherwise specified.

7½ lbs. of lath nails will be required to nail on 1000 laths.

In estimating the cost of the work only, of lathing, deduct *half the openings*—such as windows, doors, etc. Lathing is measured by the square yard. Less than half a yard is not counted. A half yard or over is counted the next above. This only applies to the final number of yards, when about to multiply by the price.

In measuring lathing and plastering, a very rapid way is to measure the length of all walls with a tape line, and take together all walls of the same height in one operation; also, the ceiling of the whole house in another. A whole house in this way may be calculated almost as easily as one room.

OPENINGS. In making deductions for openings, it will be noticed that all outside openings appear once only, and that inside doors appear twice, that is, in two rooms.

EXAMPLE.

A two story house is 24 × 36 feet inside. The first story ceiling is 10 feet, and the second story 9 feet high. The total length of the walls on the first flat 320; second flat, 360 feet. There are 4 outside doors, 3 × 7 feet; 15 windows, 3 × 6; 16 inside doors, 3 × 7 feet. How many yards of lathing are there in it?

Ceilings, 24 × 36 × 2 =	1728
Walls, first flat, 320 × 10 =	3200
" second flat, 360 × 9 =	3240
	8168

DEDUCTIONS.

4 outside doors $\frac{4 \times 3 \times 7}{2} = 42$

15 windows, $\frac{15 \times 3 \times 6}{2} = 135$

16 inside doors appearing twice $16 \times 3 \times 7 = \underline{336}$

$\underline{513}$

7655

7655 square feet $\div 9 = 850\frac{5}{9}$ square yards.

The above example applies to plastering, wall painting, kalsomining, etc., as well as lathing.

EXERCISES.

1. How much space on a wall will a lath cover, including the usual space between?

2. How much space will a bunch of lath actually cover, allowing proper space between, but nothing for waste?

3. How many laths are required for a room 10 × 15, ceiling 10 feet high? No allowance for waste.

4. How many laths are required for a room 25 × 40 feet, (ceiling 15 feet high), in which there are 4 doors, 3 × 8 feet, 6 windows, $3\frac{1}{2} \times 7$ feet? Contractors' rule. 4237

5. How many bunches should be bought in exercise No. 4?

6. How many laths are required in exercise No. 3? (Contractors' rule).

7. Suppose a house 24 × 30 feet, 16 feet high, gables 6 feet high, were lathed outside, first perpendicularly and then horizontally, as it is done in the Province of Quebec for rough-casting; the laths in both cases being placed 1

inch apart. How many laths would be required? No allowance for waste.

8. Allowing one-tenth of laths to be wasted, how many laths would be required in exercise No. 7? How many bunches should be purchased?

9. A cottage 30 × 30 feet, ceiling 10 feet high, measures 420 ft. in length of lathed walls inside. There are 2 outside doors and 6 inside doors, each 3 × 8 feet, and 6 windows, 3 × 7 feet. Find the cost of lathing at $3\frac{1}{4}$ cents per square yard.

10. A fence 45 feet long, 3 feet high, is made by putting laths 1 inch apart, diagonally, in two directions. How many laths are required, allowing one-tenth of laths to be wasted?

Plastering.

In the measurement of plastering, the same methods are used as in the previous chapter on lathing.

When the total yards in a building are found, take the nearest even yards in finding the cost. $235\frac{3}{4}$ yards would be counted as 236, etc. Less than half a yard is left out.

7 bushels of lime, 1 bushel of hair and, $1\frac{3}{5}$ cubic yards of sand will make enough mortar to first-coat 100 yards.

$2\frac{1}{2}$ bushels lime, and $\frac{3}{8}$ barrel of Plaster Paris will be sufficient to finish 100 yards.

Half the openings only are counted out.

EXERCISES.

1. How many square yards of plastering are there in the ceiling of a room 20 × 40 feet?

2. How many square yards of plastering are there in the walls of a room 28 × 18 feet, 10 feet high?

3. How many yards of plastering are there in the walls and ceiling of a room, 36 × 26, 10 feet high?

4. Allowing for an eighteen-inch base board, find the number of yards of plastering in a room 38 × 27 feet, 14 feet high?

5. Find the cost of plastering the walls and ceiling of a room, 18 × 26 feet, 12 feet high, at 25 cents per yard.

6. Find the cost of plastering a ceiling 24 × 18 feet, at 10½ cents per square yard.

7. At 12½ cents per square yard, find the cost of plastering the walls of a room 30 × 18, and 12 feet high.

8. Find the cost of plastering the walls and ceiling of a room, 18 × 12 feet, 10½ feet high, at 12 cents per square yard, making allowance for four openings, each 4 × 8 feet.

9. Find the cost of plastering a room 24 × 15 feet, 11 feet high, with 18 inch base, and having two openings averaging 24 square feet, at 10 cents per square yard, (even yards).

10. Find the cost of cementing the bottom and sides of a cistern 8 × 11 feet, 8 feet deep, at 4 cents per square foot.

11. Find the cost of lathing and plastering the ceiling of a room 25 × 32 feet, the laths costing 8⅜ cents per bunch. Putting on laths and plaster 15 cents per square yard. (Even bunches and even yards).

12. At 13½ cents per yard, find the cost of plastering the walls only of the following rooms. No allowance for openings. 30 × 20 feet, 10 feet high; 50 × 42 feet, 14 feet high; 18 × 15 feet, 9 feet high.

13. A two-story house 27 × 37 feet, with one-story kitchen attached, 20 × 30 feet, is to be lathed and plastered at 10 cents per square yard for doing the work. The ceilings are all 10 feet high; the length of the walls 760 feet. There are three outside and 13 inside doors; 3 × 8 feet, and 16 windows, 3 × 6 feet. Find the cost. 107.80

14. Find the cost of cementing a circular cistern, 8 feet in diameter, 10 feet deep, at 2¾ cents per square foot.

Find the cost, at 2½ cents per square foot, of cementing a circular cistern, 7 feet in diameter, 10 feet deep.

16. A circular cistern is 10 feet in diameter, and 4 feet deep. Find the cost of cementing at 3 cents per square foot.

17. In a two-story house, 27 × 37 feet inside, the first story is 10 feet high, the second 9 feet; the length of plastered walls on first flat 325 feet, and on second story 410 feet. There are 12 windows, 3 × 6 feet; 3 outside doors, 3 × 8 feet; and 10 inside doors, 3 × 7 feet. Find the cost at 18 cents per yard. (Even yards).

Painting, Kalsomining, etc.

The cost of painting, kalsomining, tuck-pointing, etc., is computed by the square yard.

EXERCISES.

1. Find the cost of painting a floor 33 × 50 feet, at 15 cents per square yard.

2. A close board fence 210 feet long, 7 feet high, is to be painted on one side. Find the cost at 9 cents per square yard.

3. Find the cost at 12½ cents per square yard, of kalsomining the walls and ceiling of a room 15 × 20 feet, 10 feet high. There is a base board 1 foot high around the room, and 120 feet to be deducted for openings.

4. Estimate the cost of tuck-pointing a shop-front 30 feet wide, 60 feet high, at 90 cents per square yard. 350 square feet to be deducted for openings.

5. Find the cost of painting a shingled roof, 62 feet long, each side being 28 feet wide, at 22 cents per square yard.

Papering.

Wall paper is usually 18 inches wide, and put up in single rolls of 24 feet in length, or double rolls of 48 feet in length.

The number of strips that can be cut from a roll, or double roll, will vary according to the height of the ceiling and the waste in matching.

It is almost impossible to get at the exact quantity of paper required for a room, on account of the waste in matching patterns, and the openings. The following is the rule generally used.

RULE. *Measure the distance around the room in feet. Deduct 3 feet for each window or door, and divide by 1½ feet, (the width of the paper) to find the number of strips. To find the rolls required, find how many strips can be cut from a roll and divide into the total number of strips required.*

NOTE. A part of a roll is counted as a whole roll in computing the cost.

EXERCISES.

1. How many strips of paper are required for a room 26 × 20 feet, if there are three windows and two doors?

2. How many strips are required for a room 10 × 15 ft. with two windows and one door?

3. How many strips are required for the two sides of a hall 30 feet long, if there are six doors in the sides?

4. If a ceiling of a room is 14 feet high, how many strips for the wall, can be cut from a double roll?

5. How many single rolls are required to paper a ceiling 20 30 feet?

6. How many rolls will paper a ceiling 10 × 14 feet?

7. How many single rolls are required for the wall of a room 14 × 16 feet, 12 feet high, if there are two windows and one door?

8. How many double rolls are required for the walls of a room 30 × 40 feet, 5 feet high, having in it four windows and three doors.

9. How many yards of border paper, that has two strips of the border pattern to the width, are required for the room in question, No. 8?

10. At 40 cents per double roll for the paper, and 30 cents per yard for border paper, three patterns to the width, find the cost of the paper for a room 18 × 20 feet, 9 feet ceiling, allowing for three doors and two windows.

Furnishing.

EXERCISES.

1. How many strips of carpet, 1 yard wide, are required for a room 15 feet wide?

2. How many strips of carpet, 27 inches wide, are required for a room 18 feet wide?

3. How much will be turned under or cut off from a carpet 27 inches wide, when laid on a room 15 feet wide?

4. If the strips of a 27-inch carpet are laid lengthwise on a room, 13 ft. 6 in. by 18 feet, and a half yard is lost each time for matching strips, how many yards are required for the room?

NOTE. If there are six strips of Carpet on a room, it will only be matched five times, nothing being wasted on the first strip, as there is nothing to match it with.

5. If the strips of a carpet one yard wide are laid lengthwise on a room $20\frac{1}{2} \times 28\frac{1}{2}$ feet, how many yards will be required, allowing 10 inches waste each matching.

6. Find the cost of sewing and binding the carpet in No. 5, at 5 cents per yard for binding the ends, and 3 cents per yard for sewing together.

7. Find the cost of sewing the carpet and binding the ends in question No. 4, at 4 cents per yard for seams, and 7 cents per yard for binding.

8. Find the cost of felt carpet-lining, one yard wide, at $4\frac{1}{2}$ cents per yard, for the room in question No. 5.

9. Find the cost of a stair carpet, at 90 cents per yard for a stair consisting of 16 steps, 12 inches wide, with 7 in.

risers, allowing 1½ feet to go on both upper and lower floors. 8.5

10. Find the cost of moulding for the following rooms, at 6 cents per lineal foot: (a) 12 × 16 feet; (b) 8 × 12 feet: (c) 9 × 14 feet: (d) 15 × 18 feet.

11. Find the cost of blinds for 20 windows, 42 inches wide, 7 ft. 4 in. long, at 30 cents per square yard. 16.00 17.11

12. Find the cost of curtains for four windows 8 feet high; 2 feet to be on the floor, and two curtains to each window, at 85 cents per yard.

13. At 30 cents per square foot, find the cost of oilcloth for a lobby 6 × 10 feet. 18.00

14. A room is 14 × 19½ feet. Which would be the more economical way to lay a 27 inch carpet, allowing 15 inches waste for each matching?

Stone-Work.

Building stone is sold by the cord of 128 cubic feet. Nothing less than quarter cords are counted.

A cord of stone will make 100 cubic feet when built into a wall.

Stone-work is measured by the perch of 16½ cubic feet. A piece of wall 16½ feet long and 1 foot square, makes a perch.

Cut stone, such as window sills, lintels, etc., is usually measured by the lineal foot.

In measuring mason work, for work only, the outside dimensions of the wall are taken. Thus the corners are counted twice. No allowance is made for small openings.

In estimating for material, take only the exact volume of the wall, deducting openings.

Four bushels of lime and two yards of sand will make mortar enough to lay a cord of stone, making 100 cubic feet of wall.

If the fraction of a perch is a half or more in the total of a job, make a full perch of it in estimating the cost. If less than a yard, do not count it.

EXERCISES.

1. A pile of stone 30 feet long, 15 feet wide, and 6 feet high, contains how many cords?

2. How many cords of stone in a pile 60 ft. long, 20 ft. wide, and 3 feet high?

3. What length of a wall 9 feet high and 2 feet thick, could be built from the stone in question No. 2?

4. How much lime and sand would be required to lay the stone in question No. 2?

5. How many loads of stone in a wagon box 10 feet long, 3 feet wide, and 14 inches deep, would be required to build a wall 60 feet long, 10 feet high, and 18 inches thick?

6. If a wagon box is 8 ft long, 3 feet wide, and 16 inches deep—how many loads of stone would be required for 10 cords?

7. Find the cost of building the wall in question No. 3, at 45 cents per perch.

8. How many cords of stone are required to build the foundation under a house 24 × 30 feet, (outside dimensions,) 8 feet high, 18 inches thick?

9. Find the quantities of lime and sand required in exercise No. 8.

10. Find the cost of the building in No. 8, at 40 cents per perch, (even perches).

11. A stone house is 36 × 26 feet, (outside dimensions). The cellar wall is 2 feet thick and 9 feet high. The first story 20 inches thick and 11 feet high. The second story wall 18 inches thick and 6 feet high; with two gable ends 12 feet high, and one pediment gable on side 12 feet base 8 feet high. Find the cost of the building at 50 cents per perch.

12. How many cords of stone would be required in the construction of the house in No. 11, allowing for 20 openings averaging 32 square feet surface each, 6 of them being in the cellar, 9 on the first flat, and 5 on the second flat?

13. A stone addition is 50 feet long and 35 feet wide—two side walls and one end wall to be built. The basement story is two feet thick and 9 feet high. The first story 20 in. thick and 12 feet high. Find the cost of building, (no allowance being made for openings,) at 40 cents per perch.

14. Find the cost, separately, of the stone, lime, and sand for No. 13, at $1.50 per cord for stone, 15 cents per bushel for lime, and 75 cents per yard for sand.

Brick Work.

In an ordinary wall about $\frac{1}{5}$ is mortar. When pressed bricks are used for very fine work about $\frac{1}{8}$ only of wall is mortar.

Bricks vary in size. They are usually about $8\frac{1}{2}$ inches long, 4 inches wide, and $2\frac{1}{2}$ inches thick.

Walls put on the outside of wooden buildings, (veneering,) are said to be half a brick in thickness. A wall of this kind is usually made the standard of measurement.

The following rule is found to be very accurate.

RULE. *For veneering, (half brick walls,) count 50 bricks to a square yard of surface of wall: for a wall a brick thick, 100 to the square yard: for a wall of brick and a half theick, 150 bricks to the square yard, and so on. Use outside dimensions.*

When estimating work only, half openings are sometimes deducted. When estimating material only, deduct opening.

Three bushels of lime and $\frac{2}{3}$ of a cubic yard of sand will lay 1000 bricks.

EXERCISES.

1. How many bricks are required for a wall 60 feet long 18 feet high, and half a brick thick?

2. How many bricks are required for a wall 75 feet long, 20 feet high, and one brick thick?

3. How much sand and lime are required for exercise No. 1?

4. Find the cost of laying the bricks in No. 2, at $2 per thousand.

5. How many bricks are required for a wall, 140 feet long, 21 feet high, and one and a-half bricks thick?

6. How much sand and lime are required for the wall in No. 5?

7. How many bricks are there in the walls of a building

40 × 60 feet. First story 12 feet high, and 1½ bricks thick. Second story 15 feet high, and 1 brick thick; allowing for 10 openings in lower and 6 in upper story, of 36 square feet each?

8. Find the cost of building the bricks in No. 7, at $4 per thousand.

Shingling.

The standard shingle is 4 inches wide.

If laid 4 inches exposure, a shingle covers 4 × 4 = 16 square inches. Making allowance for waste, 1000 shingles will cover 100 square feet of roof, laid 4 inches exposure.

Shingles are sold by the thousand. A bunch containing a quarter-thousand is 20 inches in width, and should contain 25 courses.

Five pounds of shingle nails are required to lay 1000 shingles.

A roof is said to be a quarter-pitch when the height is equal to one-fourth the breadth. A third-pitch when the height is one-third the breadth; and half, or square pitch, when the height of the roof is half the breadth of the building.

EXERCISES.

1. How many square feet will 1000 shingles cover, 4 inches exposure? No allowance for waste.

2. How many square feet of roof will 1000 shingles cover laid 5 inches exposure? No allowance for waste.

3. How many shingles, laid 5 inches exposure, are required to cover 100 square feet? No allowance for waste.

4. How many squares of shingles, laid 4 inches exposure, are required for a roof 40 feet long, each side is 10 feet wide, allowing waste.

5. How many thousand shingles, (squares) are required for a roof 84 feet long, one side 24 feet wide, the other 40 feet wide, laid 4 inches exposure, allowing for waste.

6. Find the cost of the nails required for No. 5, at 4c. per pound.

7. How many shingles are required to lay the roof in No. 5, if laid 5 inches exposure? No allowance for waste.

8. Find the cost of shingling a roof 64 feet long, each side 24 feet wide, 4 inches exposure, and usual allowance for waste, with shingles at $2.50 per thousand, nails at 4 cents per pound, and work of laying, at 60 cents per thousand.

Carpenter Work.

Studs in partitions are placed 16 inches apart from centre to centre, so that a 4-foot lath will cover three spaces, and nail on 4 studs.

Only a few items in carpenter work can be taken up here. Some points are already referred to under "Shingling," and other headings.

NOTE 1. In estimating the studs for a partition, one extra must be added to the number found by dividing the length by 16 inches. The first one does not count—two spaces take three studs.

NOTE 2. In matched lumber for floors, wainscot, etc., from one fifth to one-tenth the surface is lost in the "tongue and groove."

EXERCISES.

1. Allowing one-sixth of lumber to be used in matching,

how many feet are required for a floor 20 × 32 feet, 1 inch thick?

2. If one-fifth of lumber is lost in matching, how many feet are required for 280 lineal feet wainscot 4 feet high?

3. How many feet of lumber are required to lay a floor $1\frac{1}{4}$ inches thick, 30 × 50 feet, if one-eight of the lumber be lost in matching?

4. Make out an estimate, in good form, for laying a floor 40 × 45 feet, with inch matched lumber, at $16 per thousand, allowing one-eight, of lumber to be used for matching. Also 40 lbs. nails at $3\frac{3}{4}$ cents per pound; two days' work of carpenter at $1.75, and two days of laborer at $1.25, adding 10 per cent. to the total for contingencies.

5. How many studs are there in a partition 32 feet long?

6. How many feet of lumber are there in the rafters of a house 24 feet long, if the rafters are 13 feet long, and 2 × 6 inches, placed at 16 inch centres?

7. How much inch lumber is required to close in and roof a shanty 12 feet long, 10 feet wide, 8 feet high at back and 10 feet at front, the roof being made of 12 ft. lumber laid two thicknesses? No allowance for waste.

8. Make out an estimate, neatly, for replacing an old sidewalk with a new one, 40 rods long, 6 feet wide, from plank 2 inches thick, with three string pieces 4 × 4 inches under the entire length. Lumber at $9 per thousand, $2\frac{1}{2}$ ℔s nails to rod, at $3\frac{1}{4}$ cents per pound. Work as follows: Clearing away old sidewalk—three laborers 5 days at $1.10; one man and team 5 days, and another man and team 2 days, at $2.75 per day. Work laying new sidewalk—4 laborers 6 days, at $1.10; 2 carpenters 6 days, at $1.50. Add 15 per cent. for contingencies.

Fencing.

In estimating the posts for a fence, the same caution should be observed as in studs in a partition—of adding one extra. Where a fence is around a lot or field, this caution need not be observed, as each of the corner posts counts on two sides.

EXERCISES.

1. How many posts are required for a fence 36 rods long, if placed 6 feet apart.

2. How many feet of lumber are required for the above fence, if the widths of the boards, which are placed horizontally, are 12, 8, 8, 6, 7, 7, inches.

3. A common board fence, in which the following widths are used horizontally, 12, 8, 8, 7, 7, 8, is built around a field 36 × 60 rods. The posts are six feet apart, and a piece of board 6 inches wide, 6 feet long, is nailed perpendicularly opposite every place where a post appears. How many feet of lumber are required? No allowance for waste.

4. Find the entire cost of the fence in No. 3, if the lumber is $8 per thousand, the posts 10 cents each, nails $\frac{3}{4}$ ℔. to rod at $3\frac{1}{2}$ cents per pound, and the workmanship 22 cts. per rod.

5. A close board fence is made by putting on the posts a scantling 2 × 6 on top, and one 2 × 4 inches near the bottom, a board 1 inch by 12 inches all along the base, and 12 foot boards cut in two placed perpendicularly. How much lumber is required for such a fence one mile long?

6. Suppose the fence in No. 5, were made of pickets 4 in wide, 4 inches between the pickets, and that the base and pickets cost $15 per thousand, and the scantlings $10 per thousand, find the cost of the lumber.

Timber Measure.

Square timber is sold by cubic feet. To measure it is simply to find its volume in cubic feet.

Care should be taken to see that all dimensions are of the same denomination, *i. e.*, all feet, or inches, or that proper divisors are used in case some dimensions are in feet, others in inches.

EXERCISES.

1. How many cubic feet in a stick 12 × 12 inches, 30 ft. long?

2. How many cubic feet in a stick 15 × 18 inches, 60 ft. long?

3. A stick of timber is 18 × 20 inches at large end, and 12 × 12 at small end, 6 feet long. Find its volume, (Frustum of a pyramid.)

4. A stick of timber, 40 feet long, 15 × 20 inches at large end, and 12 × 15 inches at small end. Find its volume.

5. Find the cubic feet in the following sticks of timber, and the cost at 15 cents per foot:—

20	pieces	11 × 11	inches,	24	feet long ;
80	"	6 × 6	"	20	"
7	"	10 × 14	"	60	"
4	"	9 × 10	"	40	"
4	"	6 × 8	"	40	"
8	"	10 × 11	"	36	"

6. Find the cubic feet contained in the following sticks of timber:—

15	pieces	14 × 15	inches,	40	feet long ;
10	"	12 × 16	"	30	"
36	"	18 × 20	"	45	"
90	"	16 × 16	"	50	"
30	"	24 × 30	"	36	"
28	"	20 × 24	"	40	"
80	"	15 × 15	"	60	"

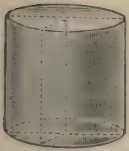
7. Find the cost of the bill in No. 6, at 16 cents per cubic foot, for all timber 40 feet long and over, and 14 cents for all under 40 feet in length.

Cisterns.

To find the capacity of cisterns in gallons, find the volume in cubic inches and divide by 277.274, the cubic inches in a gallon. Work by this method, unless otherwise specified.

A close approximate to the contents may be made by counting 25 quarts to a cubic foot.

EXERCISES.



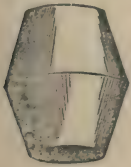
1 A circular cistern 8 feet diameter and 10 feet deep will contain how many cubic feet?

2. How many gallons will a circular cistern 6 feet diameter and 6 feet deep, contain?

3. A cistern 6 feet long, 5 feet wide, and 8 feet deep, will contain how many gallons?

4. A hexagonal cistern is 12 feet deep, the side is 7 feet. Find its volume in cubic feet.

5. A triangular cistern, 8 feet deep, has its sides 6, 8 and 10 feet respectively, Find its capacity in gallons.



6. A cistern somewhat similar in shape to the annexed engraving, is 2 feet diameter at top, 8 feet diameter at a depth of 3 feet, and 6 feet diameter at bottom, a depth of 11 feet. Find how many gallons it will contain.

Measurement of Grain.

The estimating of the quantity of grain in a bin or pile is a very useful and easy part of mensuration.

RULE. *Find the volume of the grain in cubic feet, and reduce to bushels.*

To reduce grain in cubic feet to bushels, bring cubic feet to cubic inches and divide by 2218.19—the number of cubic inches in a bushel.

A practical and useful rule is to multiply the cubic feet by 100 and divide by 128 to find bushels. This rule is almost absolutely correct.

A common rule is to multiply cubic feet by 4 and divide by 5. This, however, is not very accurate.

A heaped bushel, such as a bushel of apples, potatoes, etc., contains about 2500 cubic inches.

EXERCISES.

1. How many bushels in a bin of grain 4 × 8 feet, and 8 feet deep?

2. How many bushels in a pile of grain in the form of a pyramid, 6 feet square at the base, 15 feet high?

3. An elevator hopper is 20 feet square for 6 feet deep. The bottom part is 10 feet deep, and 2 feet square at the bottom, the lower part being in the form of the frustum of a pyramid inverted. How many bushels will it contain?

4. A circular pile of grain is 18 feet in diameter, 8 feet high. How many bushels does it contain?

Measurement of Hay.

The contents of mows or stacks of hay may be easily estimated. The accuracy will depend largely on the good judgment of the person making the estimate.

Old hay from the bottom of stacks or mows may be estimated from 15 to 17 cubic yards to the ton. Taking a mow or a stack entire, estimate from 16 to 18 cubic yards to the ton New hay, freshly put in mows or stacks, 19 to 22 yards to the ton. On loads 22 to 24 cubic yards to the ton.

EXERCISES.

1. How many tons of hay in a mow 18 × 20 feet and 24 feet deep, estimating 16 cubic yards to the ton?

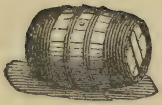
2. How many tons of hay in a new stack 50 feet long throughout; 20 feet wide at the base; 30 feet wide at a height of 12 feet; total height of stack 32 feet; estimated at 20 cubic yards to the ton? $46\frac{2}{3}$

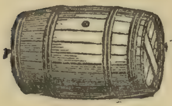
3. How many tons of hay in an old circular stack 50 ft. in circumference at the base; 75 feet in circumference at a height of 15 feet; the top in the form of a cone above this? Total height of stack 40 feet. (One ton estimated at 16 cubic yards.) [illegible]

4. A circular stack, 40 feet high, is 42 feet in circumference at 15 feet high, and 28 feet in circumference at the base. At 18 cubic yards to the ton, how many tons does it contain? [illegible]

5. A stack of new hay is 77 feet in circumference at the largest part; 21 feet from the ground, and 55 feet in circumference at the base—total height 63 feet. Find tons at 24 cubic yards to the ton.

Gauging.





By gauging is meant the estimating, the capacity of barrels, casks, etc.

On account of the variation in the shapes of barrels, it is imposible to give a rule that will measure all casks exactly. a close approximate is all that is to be expected. Note the difference in shape of the two casks illustrated.

The diameter of the barrel inside at the end is known as the **head diameter.**

The diameter at the middle is called the **bung diameter.**

Rule. *To find the average diameter, add to the head diameter two-thirds of the difference between the head and the bung diameters. Use average diameter and measure as a cylinder.*

EXAMPLE.

The head diameter of a barrel is 24 inches, the bung diameter 30 inches, and the cask is 40 inches long. Find its volume in cubic inches.

Head diameter $=24$, $+\frac{2}{3}$ of difference between head and bung diameter $=\frac{2}{3}\times 6=4$. $24+4=28$ average diameter.

$$\frac{14\times14\times22\times40}{7}=24640 \text{ cubic inches} = \text{volume.}$$

EXERCISES.

1. Find the volume of a barrel 16 inches head diameter, and 20 in. bung diameter, and 3 feet long.

2. Find the gallons in a keg 30 inches long, 15 inches head and 18 inches bung diameter.

3. A keg is 12 inches diameter at the head, 15 inches at the bung, and 18 inches long. How many gallons will it contain?

4. Find the contents in gallons, of 100 kegs of vinegar, each 24 inches long, 14 inches diameter at the head, and 16 inches at the bung.

Shoemaker's Measure.

A size of shoemaker's measure is one-third of an inch. Number one children's is $4\frac{1}{6}$ inches. There are thirteen numbers in children's wear, that is, 12 sizes above No. 1. 12 sizes = 4 inches, $4\frac{1}{6} + 4 = 8\frac{1}{6}$ inches. No. 1 in adults' measure is one-third of an inch larger than No. 13 children's footwear = $8\frac{1}{6} + \frac{1}{3} = 8\frac{1}{2}$ inches. There is a regular increase of one-third of an inch for each size.

Note. The length in the above refers to the "last" on which the boot is made—not to the length of the sole.

EXERCISES.

1. What is the length of No. 7 childrens' size?

2. What is the length of a No. 9 boot, adult's size?

3. What is the length of a No. 4 boot, children's size?

4. What is the length of a lady's shoe, No. $4\frac{1}{2}$?

5. What is the difference between a No. 10 children's size and No. 10 men's size.

6. What is the difference between No. 7 children's and No. $5\frac{1}{2}$ adults' size.

Miscellaneous Exercises.

1. How many acres more in a piece of land 4 miles square, than in one containing 4 square miles.

2. How many quarter inch cubes may be cut from a solid foot of marble? (No allowance for waste in cutting)

3. A man in plowing makes a furrow 9 inches wide. How far will he walk in plowing 2 acres?

4. Your lot of $\frac{1}{3}$ of an acre is 33 feet in width. Find its depth in yards.

5. The difference between the diameter of a circle, and the circumference, is 30 rods. How many square rods in the circle?

6. Which will fill a cistern quicker, 6 three-inch pipes or 2 six-inch pipes? Explain why.

7. A cistern is 8 ft. long, 3 ft. wide, and 6 ft. deep. How many 12 qt. pailfuls will it hold?

8. Find the value of a tree, 48 ft. long, 3 ft. diameter at small end, at 45 cents per cubic foot when squared.

9. 90 cubic yards of clay were thrown out of a cellar, $22\frac{1}{2}$ feet long, by 15 feet wide. How deep was the cellar?

10. A cask full of water weighs 126 lbs. The cask weighs 14 lbs. How many gallons of water are there? (Cubic foot of water = $62\frac{1}{2}$ lbs.

11. How many gallons will a cistern hold, the diameter being $3\frac{1}{2}$ feet, and depth 6 feet?

12. Find how many yards of cloth will be required for a circular tent 15 feet high, with a diameter of 14 feet.

13. How large a square stick may be hewn from a log 3 feet in diameter at the small end?

14. A bird hovering above a steeple which is 90 feet high, was shot by a man who was 80 feet from the foot of the steeple. He was 125 feet from the bird. How far was the bird above the steeple?

15. A cow is fastened to a stake in the centre of a circular grass plot, so that she may eat the grass off an acre and a half. How long is the rope with which she is fastened?

16. Find the volume of a pyramid 8 ft. high, standing on a square base whose edge is 3 ft.

17. A and B start from the corner of a square field of ten acres to reach the opposite corner. If both travel at the same rate, B going across and A going around, how far is A away when B reaches the point ?

18. Find the distance through the opposite corners of a cube whose height is 8 ft.

19. A field which is three times as long as it is broad, contains $4\frac{4}{5}$ acres. Find the length of it.

20. How much lumber $1\frac{1}{2}$ in. thick, is required for a lidless box, 2 ft. long, $2\frac{1}{2}$ feet deep and 1 ft. 8 in. wide inside ?

21. If a boy 4 feet high, well porportioned, weighs 60 lbs. How much will a man 6 ft. 3 in. weigh, in proportion ?

22. C borrowed from D a cubic block of hay 10 ft. long. He paid back 2 cubical blocks 6 feet square, and 4 cubical blocks 4 ft. square. How large a cube will pay the balance?

23, A penny being $1\frac{2}{3}$ inches in diametar, what part is covered by a cent one inch in diameter ?

24. Allowing 25 qts. to a cubic foot, how many gallons will pass a given point in 10 minutes, if the stream be 3 ft. deep and 20 ft. wide, and running 6 miles per hour ?

25. Two hunters, A and B, killed a deer and balanced him on a pole. A on the short arm just balanced the deer on the long arm, but B on the long arm just balanced the deer on the short arm. The weight of A and B being 192 and 147 respectively. Find the weight of the deer.

26. If 4 men were to grind off an equal portion of a grindstone, 3 ft. in diameter; how many inches would each grind off?

27. A street lamp is 24 ft. above the ground. How long a shadow will a man 6 ft. cast, if he stands 20 ft. from the lamp post?

28. Find the cost per acre of land if $56 be paid for a plot 44 ft. by 140 ft.

29. How many cubic feet of masonry in a hollow round tower, 48 ft. high, 11 ft. in diameter, if the thickness of the wall be two feet?

30. If laid 5 inches to the weather, how many shingles will be required for the roof of a building 60 by 40 feet. The roof projecting over the sides and ends 1 ft., height of roof from eave to ridge being 15 feet. No allowance for waste.

31. On a pair of scales an article put on one end seems to weigh $22\frac{2}{9}$ lbs. and on the other end 18 lbs. What is its true weight?

32. Find the cost of surveying 48 miles of railroad at $\$1.12\frac{1}{2}$ per chain.

33. A field in the form of a rectangle is 15 chains long, and 40 rods wide. How many acres does it contain?

34. How many loads of gravel will be required for a road $3\frac{1}{2}$ miles long, if it is spread 9 ft. wide and 8 in. deep?

35. Find the cost of fencing $\frac{1}{4}$ acre in the form of a rectangle, at 50 cents per rod, if it is 33 ft. wide.

36. Find the number of feet of lumber in the following: a stick of timber 12 × 12 inches, 48 ft. long; a stick 9 × 9 inches, and 27 ft. long.

37. Find the number of perches of stonework in a wall under a building 20 ft. wide and 30 ft. long, $7\frac{1}{2}$ ft. high, $1\frac{3}{4}$ ft. thick, allowing for 2 doors, $7 \times 3\frac{1}{2}$ feet, and 2 windows 3 × 3 ft.

38. Find the number of bushels in a bin 6 × 6 feet and $4\frac{1}{2}$ feet deep.

39. Find the diameter of a cistern 6 ft. deep, which will contain 1572 gallons.

40. Find the cost of carpeting a room 18 × 12 ft. with carpet 27 inches wide, at $1.80 per yard.

41. A contractor undertakes to dig a ditch $3\frac{1}{2}$ miles long, 12 feet wide at top, and 6 ft. at bottom, and 12 feet deep at 18 cents per cubic yard. How much did he receive for the work?

42. Find the cost of plastering a hall 40 ft. long, 24 ft. wide, 5 ft. high, allowing for 4 doors $3\frac{1}{2}$ ft. × 8 ft., and 12 windows 3 ft. × 7 ft., at 24 cents per square yard.

43. Find the number of gallons contained in a cistern 8 ft. in diameter and 6 feet deep.

44. Find the number of cords of stone required to build a semi-circular wall measuring 90 feet in length, around the outside, 12 feet high, and 2 feet thick.

45. Find the number of bushels contained in an elevator bin of the following dimensions:

The top part is a regular parallelopipedon 8 ft. square at top, and 6 feet deep. It then slopes inward, and at the bottom is only 2 ft. square. The bottom part being 4 feet deep.

46. Define Hexagon, Parallelopipedon, Cube, Multilateral figure, Frustum of Cone.

47. Give the rules for finding the volume of three different kinds of solids, and exemplify each by two problems, worked out.

48. Find how many cubic yards of sand there are in a heap 35 feet in circumference, and 5½ feet high.

49. Find the cost of digging a ditch 500 yards long, 4 feet deep, 4½ feet wide at top, and 3 feet wide at bottom, at 30 cents per cubic yard.

50. How many feet of lumber, ½ inch thick, can be sawn out of a log 12 feet long, 3 feet diameter at large end,, and 2 ft. 9 in. at top. Making no allowance for waste except the slabs.

51. Find the estimated cost of building a side walk 300 feet long, 12 feet wide, there being 36 days' work of men at $1.25 per day, 10 days of men at $1.50, 6 days of foreman at 2.00 per day, 3 days of man and team at $3.00 per day, 300 lbs. nails at 3¼ cents per lb, the plank to be two inches thick, and four string pieces under it the entire length, 4 × 4 inches. Lumber all to be $12.00 per thousand.

52. Find the cost per cubic yard for digging an L shaped cellar, 6 feet deep, if $21.60 were paid for the job. Dimensions are as follows, taking the sides in order beginning with the base of the L;— 30, 24, 15, 12, 15, 12.

53. How many acres in 1600 feet of street 100 feet wide?

54. A ½ acre lot is 210 feet in depth. Find its frontage.

55. How many cubic feet, etc., will a wagon-box 10½ feet long, 3¼ feet wide, and 11 inches deep, contain.

56. How many gallons in a trough-shaped cistern 7 feet deep, the top being 8 × 12 feet, and the bottom 6 × 8 feet, the bottom and top being rectangular parallelograms.

57. I want to lay a carpet 27 inches wide to the best advantage on a room 16 by 19 feet. What direction will the strips run, and how many yards will it take?

58. A barn is 40 by 60 feet, the posts 20 feet high. The roof projects 1½ feet over the end and sides. The roof is 10 feet high above the walls. How many feet of inch hemlock are required to close in the building and sheet the roof?

59. Find the cost of excavating, a circular fish pond 60 feet diameter at top and 30 feet at bottom, 4 feet deep, at 25 cents per cubic yard.

60. There is a conical pile of grain on a floor, 6 feet deep, 10 feet in diameter. How many bushels does it contain? ($\frac{100}{128}$ method.)

61. Find the cost of lathing the walls and ceiling of an **L** shaped house of the following dimensions :—the walls of first story are 520 feet long, and walls in second story 600 feet long. First story 11 feet high, second story 9 feet high. Cost of lathing $2\frac{1}{2}$ cents per yard. The sides of the house taken consecutively being, 50, 40, 25, 20, 25, and 20 feet.

62. Find the number of gallons contained in a circular tank, $5\frac{3}{5}$ feet in diameter at bottom, and $4\frac{9}{10}$ feet diameter at top, and 4 feet high. (Exact answer required.)

63. Find the cost of laying a floor of matched lumber in a house 30 × 40 feet, and kitchen 15 × 15 feet. and two semi-circle bow windows 6 ft. 5 in. in diameter, lumber to be $1\frac{1}{4}$ inch thick, and costing $30.00 per thousand.

64. Find the cost of covering a round church steeple, at 25 cents per square foot, if the steeple is 12 ft. 3 inches diameter at the base, and 15 ft. high. On top of the steeple is a ball one foot in diameter, which is also to be covered.

65. Find the cost of fencing a plot of ground in the shape of a right-angled triangle, the base of which is 160 feet, and perpendicular 120 feet. The posts are to be placed 10 feet apart and cost 8 cents each. Two scantlings, 2 × 4 inches are nailed on the posts; and a board 12 inches wide runs all around the base. The pickets are 3 inches wide and 3 inches apart, and are $3\frac{1}{2}$ feet high. Lumber costs $32.00 per thousand; other charges as follows: 100 ℔s. nails at $3\frac{1}{2}$ cents per ℔; and man at work for one week at $1.75 per day.

66. Find the area in yards of an elliptical plot of ground the greatest diametrical length being 150 feet, and the smallest 84 feet.

67. Find the number of perches of stone work in the walls of a round tower, the outer diameter of which is 30 feet, inner diameter 24 feet, height 60 feet.

68. Define Circle, Radius, Hypothenuse, Pyramid, Frustum, Rectangle, Parallelogram, Sphere, Sector, Rhombus.

69. How many square rods, etc., are contained in an elliptical race course 4 rods wide, the long and short diameter of the plot inside the track being 60 and 40 rods respectively.

70. A circular tower has a solid foundation of masonry, 60 feet in diameter, 18 feet high. It is also 10 feet under ground. Find the cost of excavating for it at 15 cents per cubic yard, and the cost of building it at $1.25 per perch.

71. How many cords of stone are required for the foundation in No. 70, and how many yards of sand and bushels of lime?

72. Find the cost of constructing a sidewalk 2 miles long, on both sides of a street if it is to be 10 feet wide, and to have 5 supports underneath of timber 3 by 6 inches the entire length; the planking to be two inches thick; 3 lbs. of spikes required per rod, at $3\frac{1}{4}$ cents per pound; labor as follows: 2 men for 12 days, at $1.75 per day, 10 men for 12 days, at $1.20; and 3 teams for 2 days, at $3.00 per day; lumber $16.00 per thousand.

73. A round stick of timber is 14 inches diameter at small end and 28 inches at large end, and 100 feet long. How many cubic feet does it contain?

74. How many feet of lumber are required to sheet the roof of a building 80 × 100 ft, the ridge being 30 feet high above the plates; and how many shingles are required to cover the roof at 4 inches exposure? Usual allowance for waste.

75. The following are the breadths in inches of boards 14 feet long and $1\frac{1}{2}$ inches thick:—13, 12, 12, 14, 16, 11, 10, 9, 9, 9, 8, 8, 12, 10, 15, 15, 15, 14, 14, 15, 16, 9, 6, 6, 12, 12, 13, 13, 13, 14, 14, 14, 15. Find how many feet of lumber, and the value at $16.50 per thousand.

76. How many lath are required to cover the outside of a house, put on in the usual way, estimated by the practical

rule, if the house is 38 feet long, 30 feet wide, 20 feet high, and the gable ends 10 feet high, no allowance being made for openings?

77. Which would be the least expensive direction to lay the strips of a 27-inch carpet on a room 12 × 16 feet; and find the cost of the carpet at 90 cents per yard, allowing 14 inches waste for matching every strip except the first?

78. Find the length and breadth in rods of a rectilinear field twice as long as it is wide, containing 20 acres.

79. Find the area in acres of a field the dimensions of the sides being, 40, 50, 30, and 30 rods, read consecutively. A line joining two opposite corners being 30 rods, this line forming an equilateral triangle with the two 30 rod sides.

80. Find the area in acres of a circular piece of land one mile in circumference.

81. Find how many bricks are in a wall 120 feet long, 18 inches thick, and 6 feet high, allowing $\frac{1}{5}$ for mortar, the bricks being 9 by $4\frac{1}{2}$ by $2\frac{1}{4}$ inches.

82. Find the cost of excavating the foundation for a house on a hillside, 30 by 40 feet, 6 feet deep at the front end, and $3\frac{1}{2}$ at the rear end, at 18 cents per cubic yard.

83. State two rules for finding the area of triangles. Give an example of each worked out.

84. State the rule for finding the contents of a frustum of a pyramid or cone, and work an example, one of each illustrating your rule.

85. State the rule for finding the contents of a cone and a pyramid, and work cut an example of each.

86. How many gallons of water are there in a circular cistern, 10 feet deep and 8 feet diameter?

87. How many bushels of grain are there in a pile on a barn floor, in the form of a cone 6 feet high, 30 feet in circumference. Use $\frac{100}{128}$ method.

88. How many yards of plastering are there in a two-story house, 28 by 38 feet, the ceiling of first story being 10 feet high, and the walls measuring 310 feet in length, and the second story ceiling 9 feet high, and the walls being 380 feet long. There are three outside doors 3 by 8 feet, 12 inside doors 3 by 8 feet, and 14 windows 3 by 7 feet.

89. How many gallons of water (exact gallons) in a circular pan 15 inches deep, 14 inches in diameter at bottom, and 21 inches diameter at top.

90. How many bushels of grain are contained in a heap in the form of a pyramid 10 feet square at the base, and 6 feet high. $\frac{100}{128}$ method.

91. Find the diameter of a circular flower plot in a garden containing 100 square yards.

92. If a cannon ball, 3 inches in diameter, weighs 40 ℔s., how much would one weigh, 1 foot in diameter, made of the same metal ?

93. If a triangular field, having sides 30, 40 and 50 rods, were cut in two parts by a line, meeting each of the 30 and 40 sides in the middle, what would be the area of each part of the triangle in acres.

94. Find the value of a load of 4-foot wood, piled 3 ft. 8 inches high, on a sleigh rack that is 11 ft. long, at $2.75 per cord.

95. How many feet of lumber are required for 280 feet of close board fence made 5 feet high, with 2 pieces the en-

tire length of it 2 × 4 scantling, and posts made of 4 × 5 in. timber, 8 feet long and set 7 feet apart?

96. At 18 cubic yards of hay to the ton, how many tons are there in a mow, 18 × 20½ feet, 15 feet high, and what is its value at $6.75 per ton?

97. A building is 30 feet wide and the roof 12 feet high above the building. Find the length of the rafter.

98. How many shingles, laid 5 inches exposure, would be required to cover the roof in No. 97, if it projected 18 inches over the ends and sides of the building, which is 40 feet long? (Usual allowance for waste).

99. A spherical shell, 2 feet diameter, made from material weighing 840 lbs. per cubic foot, weighs 2530 lbs. Find its thickness.

100. Find the distance through the opposite corners of a cube whose length is 8 feet.

101. A field which is three times as long as it is broad, contains 10⅘ acres. Find the length of it.

102. A and B start from the corner of a square field of ten acres to reach the opposite corner. If both travel at the same rate, B going across and A going around, how far is A away when B reaches the point?

103. A teamster agreed to cart from depot, 2½ miles of cast iron water pipe, 12 inches inside diameter and 1 inch thick in 12 foot lengths, the flange at every joint being 6 inches long, 1¼ thick, and 15 inches inside diameter, at 6½ cents per hundred. What would his bill be if a cubic foot of metal weighs 800 lbs.

Short Rules on Circles, etc.

We give these short, handy rules for accurate measurements in tabular form for reference. They may be cut out and pasted in a memorandum book and carried in the pocket.

1 Diameter of circle × 31.416 = circumference.
2 Radius of circle × 6.283185 = circumference.
3 Square the radius of circle × 3.1416 = area.
4 Square the diameter of circle × 0.7854 = area.
5 Square the circumference of circle × 0.07958 = area
6 Half the circumference of circle × half its diameter = area.
7 Circumference of circle × 0.159155 = radius.
8 Square root of area of circle × 0.56419 = radius.
9 Circumference of circle × 0.31831 = diameter.
10 Square root of area of circle × 1.12838 = diameter.
11 Diameter of circle × 0.86 = side of inscribed equilateral triangle
12 Diameter of circle × 0.7071 = side of inscribed square.
13 Circumference of circle × 0.225 = side of inscribed square.
14 Circumference of circle × 0.282 = side of an equal square.
15 Diameter of circle × 0.8862 = side of an equal square.
16 Base of triangle × ½ altitude = area.
17 Multiplying both diameters and .7854 = area of an ellipse.
18 Surface of sphere × ⅙ its diameter = solidity.
19 Circumference of sphere × its diameter = surface.
20 Square the diameter of sphere × 3.1416 = surface.
21 Square of circumference of sphere × 0.3183 = surface.
22 Cube the diameter of sphere × 0.5236 = solidity.
23 Cube of radius of sphere × 0.5236 = solidity.
24 Cube of circumference of sphere × 0.016887 = solidity.
25 Square root of surface of sphere × 0.56419 = diameter.
26 Square root of surface of sphere × 1.772454 = circumference.
27 Cube root of solidity of sphere × 3.8978 = circumference.
28 Cube root of solidity of sphere × 1.2407 = diameter.
29 Radius of sphere × 1.1547 = side of inscribed cube.
30 Area of its base × ⅓ its altitude = solidity of a cone, or pyramid, whether square or triangular.
31 Area of one of its sides × 6 = surface of a cube.

ANSWERS.

Page 11.

1. 16, 20
2. 27, 25
3. 3, 7
4. 5, 6
5. 24, 9
6. 81, 15
7. 21, 42
8. 54, 33

Page 13a.

1. 33
2. 93·786
3. 25
4. 45
5. 95
6. 75
7. 85
8. 125
9. 179
10. 121
11. 109
12. 115
13. 307
14. 294
15. 312
16. 216
17. 351, 714
18. 702
19. 997
20. 1432
21. 5464
22. 19683
23. 2732
24. 1994

Page 13*b*.

1. $\frac{5}{16}$
2. $\frac{13}{14}$
3. $\frac{25}{48}$
4. $\frac{24}{135}$
5. $\frac{37}{40}$.
6. $5\frac{1}{2}$
7. 4.213
8. $8\frac{1}{3}$
9. $\frac{75}{111}$
10. $\frac{24}{31}$
11. $\frac{121}{333}$
12. 95,353

Page 13*c*.

1. .25
2. ·33
3. ·035
4. .98
5. ·716
6. .216
7. 7·924
8. 1.4142
9. 1.73205
10. 14.004 +
11. 3.00001 +
12. 20.78
13. 11.2101
14. 37·5
15. .9128
16. .9354
17. 1.5411
18. 15·7306

Page 15.

1. 4
2. 6
3. 25
4. 55
5. 78
6. 106
7. 97
8. 504
9. 145
10. 364
11. .45
12. .046
13. 2.13
14. 6.54
15. 2.34
16. .25
17. 758
18. $\frac{10}{11}$
19. $\frac{6}{35}$
20. $\frac{24}{25}$
21. $3\frac{2}{5}$
22. 1.42 +

23. $2\frac{1}{4}$
24. 26.509 +
25. 5.16 +
26. 1494.2 +
27. 1 25962 +
28. 1.44224
29. 8.572 +
30. 206.6016 +

Page 28.

1. 13 a., 20 rods
2. $53\frac{1}{3}$ sq. yds.
3. $366\frac{2}{3}$ sq. yds.
4. 15 a.
5. 10 a. 1100 sq. yds.
6. 600 a,
7. $28\frac{8}{9}$ yds.
8. $105\frac{5}{9}$ yds.
9. 181 a. 3960 sq. yds.

Page 29.

1. 44 yds. long.
2. 400 rds. long.
3. $24\frac{7}{16}$ rds.
4. $18\frac{9}{14}$ ft.
5. $2\frac{2}{3}$ yds.
6. 25 ch.
7. 40 rods.
8. 51.688 + ft.
9. 101. 18 rds.
10. 40 rds 120 rds.

Page 31.

1. 25
2. 53.814 +
3. 137.295 +
4. 10 ft. 11.48 in.
5. 13 ft. 10 811 in.
6. 104 ft. 10 in.
7. 67 ft. 10.07 in.
8. 22 ft. 3.28 in.
9. 96.2 ft.

Page 33.

1. 528 ft.
2. 180 ft.
3. 5 ft. 4. 49 in.
4. 3 ft. 8 in.
5. 11.324 yds.
6. 1 mi. 37.77 rds.
7. 108.16 + rods
8. $133,625
9. 1 mi. $245\frac{5}{8}$ rd.
10. 210 ft.
11. 51 ft. 5.03 in.
12. 18 ft. 8.8 in.
13. 40 ft. 4 in.
14. 21.23 + ft.

Pages 36—37.

1. 36 sq. ft.
2. 18 sq. ft. 31 sq. in.
3. 20 sq. yds. 1 sq. ft. 72 sq. in.
4. 67 a. 2982 sq. yds 2 sq. ft.
5. $4\frac{1}{2}$ a.
6. 2400 sq. ft.
7. 96 sq. ft.
8. 384 sq. ft.
9. 17100 sq. ft.
10. 7 a. 80 sq. rods.
11. 1040 sq. ft.
12. 30 a.
13. 80 ft.
14. 14 a. 10 sq. rds.
15. 1620 sq. ft.
16. $31\frac{1}{4}$ a.

Pages 38—40.

1. 150
2. 176.156
3. 76.681
4. 10.79.
5. 9.921
6. 30.195
7. 1054352.08 sq. yds.
8. 18060 sq. yd.
9. 5515650 sq. ft.
10. 2310, 6930
11. 7 sq. ft. $18\frac{2}{5}$ sq. in.
    21 sq. ft. 56 sq. in.
    35 sq. ft. $93\frac{1}{3}$ sq. in.

12.
13.
14.

Pages 40—41.

1. 1 a. 99.8 sq. rods.
2. 310.444 + sq. ft.
3. 779.41 sq. yd.
4. 84.44 a.

Page 42.

1. 2160 sq. ft.
2. 6 a.
3. 75 a.
4. 20 rods.
5. 40 rods.

Pages 43—44.

1. 1650 sq. ft.
2. 20 a.
3. 10 sq. ft.
4. 20 sq. ft.
5. 5 a.

Page 46.

1. 132 ft.
2. $3\frac{1}{7}$
3. 396 yds.
4. $43.20
5. $25142\frac{6}{7}$
$25132\frac{4}{5}$

Pages 46—47.

1. 28 ft.
2. 77 ft.
3. 283 yds. $1\frac{9}{11}$ ft.
4. 7ft.; $7\frac{88}{31416}$
5. 84 rds.
6. 70 mls.

Pages 47—48.

1. 5544 sq. ft.
2. 154 sq. yds.
3. $44.55.
4. $69.30.
5. 246400.
246301.44.
6. $8662\frac{1}{2}$ sq. ft.
7. $1.54.
8. 27.72.
9. 231 sq. in.
10. $3\frac{1}{7}$; 3.1416.
11. $6160.
12. $96\frac{1}{4}$.

Page 49.

1. 280 ft.
2. 21 ft.
3. 40 rds.

Page 50.

1. 770 sq. rds.
2. 9 a. $131\frac{3}{7}$ sq. rds.
3. $6285\frac{5}{7}$ sq. rd.
4. (a)$1124\frac{2}{7}$ sq. rd. = 7 a. $4\frac{4}{7}$ sq. rd.
(b) 960 sq. rd. = 6 a.

Pages 51—2

1. $962\frac{1}{2}$ sq. rds.
2. 1350 sq. ft.
3. 6.547 a.
4. 924 sq. ft.
5. 21.4 ft.
6. 1280 sq. ft.
7. 7200 sq. ft.
8. $143\frac{2}{11}$ deg.
9. $57\frac{3}{11}$ deg.
10. $4\frac{7}{32}$ a.
11. 120 deg.

Pages 53—4.

1. 14.142
2. 22.627 +
3. ~~15.6~~ sq. rds.

Page 55.

1. $471\frac{3}{7}$ sq. ft.
2. $165942\frac{6}{7}$ sq. ft.
3. 69696 sq. ft.

Page 56.

1. 7260.
2. 16 a. 80 rds.
3. 4466 sq. yds.

Page 57.

1. 66 ft.
2. 187 ft.

Page 58

1. 169.842 sq.ft.
2. 376 sq. ft.
3. 619.056 sq. ft.

Pages 58—9

1. $314\frac{2}{7}$
2. $2011428571\frac{1}{7}$
3. $2245\frac{1}{11}$
4. 2464

Page 59.

1. 1440 sq. ft.
2. 35 sq. yds.
3. 220 sq. yds.

Pages 60—1.

1. 192 sq. ft.
2. 288 sq. ft.
3. 2 a. 52 sq. yds.

Page 62.

1. 512 cu. ft.
2. 125 cu. ft.
3. 125 cu. yds.
4. 24 ft.
5. 8 pieces.

Pages 63—4

1. 160 cu. yds.
2. $60.
3. $10\frac{15}{16}$ cu. ft.
4. 78 cu. ft. 810 cu. in.
5. 384

6.
7. 7862 ½ gals
8. 4 ft. 6 in.
9. 4 ft

Pages 65—6

1. 48 cu ft
2. 187½ cu in
3. 194 cu. ft. 363 cu. in.
4. 3 ft. 8 in.
5. 748.244 cub in.

Pages 66—7

1. 11 cub. ft.
2. 440 cub. ft.
3. 91.63 cub. ft.
4. ~~384~~ cub. ft.
5. $21\frac{3}{14}$ cub. ft.
6. $141\frac{3}{7}$ cub. in
7. 13 miles $4578\frac{2}{11}$ ft.
8. 254544 ft.
9. 8250 cub. ft.
10. $235\frac{5}{7}$ cu. in.

Page 68

1. 48⅛ cub. in.
2. 3267 cub. in.
3. $236\frac{25}{28}$ cu. in.
4. $5443\frac{3}{7}$ cu. in.

Page 69

1. 924 cub. in.

Pages 69—70

1. 1½ cub. ft.
2. $4\frac{3}{8}$ in.

3. 35.361 cu. in.
4. ~~47.95~~ cub. in.

Pages 71—2

1. $1005\frac{5}{7}$ cu. ft
2. 960 cub. ft
3. 19.6 cub. f
4. 1800 cub. f
5. 39 cub. ft. 608 cub. in.
6. $235\frac{5}{7}$ cub. ft.
7. 467 64 cu. ft.
8. 528.8 + bus.
9. 20565.9 gals.
10. 3968626 cub. ft.
11. $3466145\frac{31}{81}$ cub. yds.
12. 1408.157 cu. ft.
13. 1248 cub. ft.
14. 180 cub. ft.

Pages 73—4

1. 780 cub. ft
2. 556 cub.
3. 10780 cu.
4. 304 cub. ft.
5. 76.2 cub. ft.
6. 2389.5 gal.
7. 3120 cub. ft.
8. 5501.9 gals.
9. 280 cub. ft.
10. 10.99 tons.

Page 75

1. $905\frac{1}{7}$ cub. in.

2. 1,462,114,401,523,809,523,809$\frac{1}{21}$ cu. yds.
3. 14$\frac{1}{7}$ cub. ft.
4. 1767$\frac{6}{7}$ cu. in.
5. 2145.524 cu. in.
6. 26.524 lbs.
7. 185.55 lbs.
8. 923$\frac{4}{7}$ lbs.
9. 9 55 ft. dia.
10. 90029.761 lbs.
11. 12.04 ft.

Pages 80—81

1. 11.547 mls.
   4.782 mls.
   3.671 mls.
2. 108 ft

Pages 82--84

1. 2½ cords.
2. 7½ cords.
3. 14 cords.
4. 129 cords 88 cub. ft.
5. 21 cords 84 cub. ft.
6. $356.65.
7. 4$\frac{4}{11}$ ft.
8. 7$\frac{1}{5}$ ft.
9. 5$\frac{1}{19}$ ft.
10. 3$\frac{7}{19}$ ft
11. 37$\frac{13}{16}$ cords.
12. 112½ cords.
13. 60 loads.
14. $271.69.
15. $4.69.
16. 12$\frac{4}{5}$ ft.
17. 21$\frac{1}{3}$ ft.
18. 7$\frac{17}{25}$ ft.
19. 1$\frac{61}{64}$ cords.
20. 21$\frac{1}{2}$ in.
21. 16½ cords.
22. $3.12

Pages 85—87

1. 144 ft.
2. 24 ft., 36 ft., 42 ft.
3. 12 ft.
4. 60 ft.
5. 720 ft.
6. 933$\frac{1}{3}$ ft.
7. 256 ft.
8. 300 ft.
9. 6753$\frac{1}{3}$ ft.
   $49.05.
10. 224 ft.
11. 252½ ft.
12. 1200 ft.
13. 987$\frac{3}{7}$ ft.
14. 31.
15. 576 ft.
16. $14.48.
17. $1257.795.
18. 4200 ft.
19. $77.86.
20. 84 ft.
21. 80 ft.
22. (*a*) 24025 ft.
    (*b*) $210.02
23. $561.79

Pages 88—89

1. 160 rods sq.
2. $11520.
3. $1.50
4. 576
5. 7½ a.
6. 40 a.
7. (*a*) 15 a.
   (*b*) nearly 49 rods.
8. 3a. 136 rods
9. 22.413 a.
10. 266$\frac{2}{3}$ rds.
11. 51$\frac{6}{7}$ ft.
12. 3$\frac{17}{20}$ a.
13. 33.47 rds.
14. 1.001 a.

Pages 91—92

1. 90 sq. in.
2. 31¼ sq. ft.
3. 1040 laths
4. 5234.
5. 105
6. 24 bunches
7. 4493
8. (*a*) 4993
   (*b*) 100 bunches
9. $17.58.
10. 360.

Pages 92—94

1. 88$\frac{8}{9}$
2. 102$\frac{2}{9}$
3. 241$\frac{7}{9}$

4. $294\frac{5}{9}$
5. $42.25
6. $5.04
7. $16.00
8. $10.44
9. $11.70
10. $15.68
11. $15.98
12. 62.55
13. $107.90
14. $8.31
15. $6.46
16. $6.12
17. $171.72

Pages 94—5

1. $27.50
2. $14.70
3. $11.25
4. $145
5. $84.87

Page 93

1. 52 strips.
2. 28 "
3. 28 "
4. 3 "
5. 20 rolls.
6. 3 double or 5 single rolls
7. 17 rolls
8. 9. rolls.
9. $23\frac{1}{3}$ yds.
10. $6.30

Pages 97—8

1. 5 strips.
2. 8 "
3. 9 in.
4. 38½ yds.
5. $68\frac{1}{6}$ yds.
6. $2.41
7. $1.83
8. $2.99
9. $8.~~68~~ 50
10. $12.48
11. $15.98
12. $22.66
13. $24.00
14. 48 yds. if laid lengthwise or $45\frac{1}{3}$ when laid across.

Pages 99—100

1. $21\frac{3}{32}$ cds.
2. $28\frac{1}{8}$ cds.
3. 156¼ cds.
4. 56¼ yds. sand.
112½ bus. lime
5. 33 loads
6. 40 loads
7. $76.70
8. $12\frac{6}{25}$ cds.
9. $24\frac{12}{25}$ yds. sand.
$48\frac{24}{25}$ bus. lime
10. $31.60
11. $186.50
12. 50.573 cds.
13. $198.00
24. $122.51, $49, $122.51

Pages 101—2

1. 6000
2. 16666
3. 4 yds. sand
18 bu. lime
4. $33.33
5. 49000
6. $32\frac{2}{3}$ yds. sand
147 bu. lime
7. 64934 bricks
8. $59.74

Pages 102—3

1. $111\frac{1}{9}$ sq. ft.
2. $138\frac{8}{9}$ sq. ft.
3. 720
4. 8 thousand
5. $53\frac{19}{25}$
6. $10. 75
7. 38707
8. $101.38

Pages 103—4

1. 768 feet
2. 1400 feet
3. 2143 feet
4. $44.45
5. 25 studs
6. 494 feet
7. 684 feet
8. $205.21

Page 105

1. 100 posts
2. 2376 feet
3. 14784 feet
4. \$218.35
5. 45760 feet
6. \$404.80

Pages 106—7

1. 30 feet
2. $112\frac{1}{2}$ feet
3. 10.18 feet
4. 197 88 cu. ft.
5. 1585 feet
\$237.75
6. $29958\frac{1}{3}$ feet
7. \$$4677.33\frac{1}{3}$

Page 107

1. 502.85 cu. ft.
2. 1057.67 gals.
3. 1495 7 gals.
4. 1527.12
5. 1196.56
6. 2343.86 gals.

Page 108

1. 200 bu.
2. $140\frac{5}{8}$ bu.
3. 3031 25
4. $530\frac{5}{14}$

Page 109

1, 20 tons
2. $55\frac{5}{9}$ tons
3. 19.56
4. 5 43
5. 21.522

Pages 110—11

1. 5 cu ft. 121[illegible] cu. in.
2. 24.5 gals.
3. 9.9 gals.
4. 1332.4 gals.

Page 111

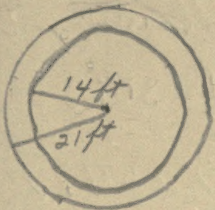
1. $6\frac{1}{6}$ in.
2. $11\frac{1}{6}$ in.
3 $5\frac{1}{6}$ in.
4, $9\frac{4}{6}$ in.
5. $4\frac{1}{3}$ in.
6. $5\frac{5}{6}$ in.

Pages 112—22

1. 7680 acres
2, 110592 q'ts
3. 22 miles
4 $146\frac{2}{3}$ yds.
5. 154 sq. rds.
6 6 inch pipes
7. 30[illegible] pails
8. \$97 20
9. $7\frac{1}{5}$ feet
10. $11\frac{1}{5}$ gals.
11. $360\frac{15}{16}$
12. 40.46 sq. yds.
13. 25.4 inches
14. 6.04 feet
15 48.06 yds.
16. 24 cub. ft.
17. 23.44 rods
18. 13.05 feet
19. 48 rods long
20. $35\frac{25}{32}$ feet
21. 228.881 lbs.
22 6.78 + ft side
23. $\frac{25}{49}$
24. 1980000 gal.
25. 168 lbs.
26. 2.42 in.
2.86 "
3.72 "
9 "
27. $6\frac{2}{3}$ feet
28. \$396.00
29. $2715\frac{3}{7}$ cu. ft.
30. 23213 shingles
31. 20 lbs.
32. \$4320
33. 15 acres-
34. $4106\frac{2}{3}$ loads
35. \$22.00
36. $758\frac{1}{4}$ feet
37. 66.9
38. 126.2 + bush
39. 7.3 + ft.
40. \$57.60
41. \$13305.60
42. \$37.81
43. 1880.28 gal.
44. 20.845 cords
45. 386.39 bus.
48. 6.616 cu. yd.
49. \$250.00
50. 1089 feet
51. \$196.35
52. 18 cents
53. 3 673 acres
54. $103\frac{5}{7}$ feet

55. 31.28 cu. ft.
56. 3081.01 + gal
57. lay crosswise
48 yds.
58. 7406.36 feet
59. 51.11$\frac{1}{9}$
60. 122.77 bus.
61. \$39.22$\frac{2}{9}$
62. 540.6 gals.
63. 54.65
64. \$72.97
65. \$80.56
66. 1100 sq. yds.
67. 925$\frac{5}{7}$
69. 678.857
70. \$6000
71. 792 cords.
3168 yds. of
sand
1584 bus. of
lime
72. \$9621.60
73. 249.53 cu. ft.
74. 10000 ft lum
100,000 shg.
75. 11 49
76. 5584 laths
77. lengthwise
\$30.55
78. 40 rds brdth.
80 rds length
79. 6.18 acres
80. 50 90
81. 16,384 bricks
82. \$38.00
86. 3133.805$\frac{5}{7}$ gl.
87. 111.86 bush
88. 908$\frac{5}{9}$ sq yds
89 13.1907 gal
90. 156¼ bus.
91. 11.28 yds.
92. 2560 lbs
93. 2$\frac{13}{18}$ acres
94 \$3.47
95. 2320feet
96. 11$\frac{7}{18}$tons
\$76.88
97. 19 20 + feet
98 14248 shing
99. 4.14 inches
100 1385 feet
101 72 rods
102 23.44 rods

To find the area of a ring; add th
two radii together and multiply by the
difference between them and then by 3
thus;



14 + 21 = 35
21 − 14 = 7
245
$3\frac{1}{7}$
35
735
770 sq ft.

To find the area of any regular figure multiply the square o side by the appropriate decimal in the following table

3	sided	figure	.433013
4	"	"	1.
5	"	"	1.720477
6	"	"	2.598078
7	"	"	3.633912
	"	"	4.828427
	"	"	6.181824
0	"	"	7.694209
	"	"	9.365640
2	"	"	11.196152

o find Diagonal of parallelopipedon quare the length, the width, and he height add the products ogether and extract the square oot.

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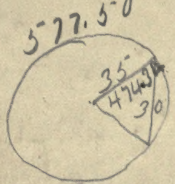
sq root of 3 = 1.7320508
" " " 2 = 1.41421356237
area of sq in circle = $\frac{7}{11}$ of area of circle

To find the area of an equilateral triangle take $\frac{1}{2}$ side square it and multiply by $\sqrt{3}$
sq root of 7 = $2.6\overset{45}{\cancel{64}}75$
1000 oz of water = 1 cu ft or $62\frac{1}{2}$ lbs or 25 cu. inches ~~sq. inches~~



area of circle = 577.50
area of triangle 474.34
area of — 103.16